# Workshop
# Topology and Computer 2016

A workshop "Topology and Computer 2016" will be held as follows. This Workshop is supported by Grant-in-Aid Scientific Research (B) No. 15H03619 (Sadayoshi Kojima, Tokyo Institute of Technology) and others.

Date: October 28 – 30, 2016
Venue: Lecture hall at College Plaza
Address: Meitoku-kan building, 1-51, naka-doori 2 cho-me, Akita City, Akita, Japan
Web: http://www.math.akita-u.ac.jp/tc2016/index-e.html

## Program

**October 28**

13:30 – 14:15 Yuichi Kabaya (Kitami Institute of Technology)
Optimal Kazhdan constants and semidefinite programming

14:30 – 15:15 Genki Kusano (Tohoku University)
Statistical analysis of persistent homology

15:30 – 16:15 Hiroki Kodama (University of Tokyo)
Protein structure analysis and $SO(3)$

16:30 – 17:15 Szilárd Zsolt Fazekas (Akita University)
Regular languages closed under word operations

17:30 – 18:00 Masaaki Suzuki (Meiji University)
Epimorphisms between two-bridge knot groups and their crossing numbers

**October 29**

9:30 – 10:00 Fumikazu Nagasato (Meijo University)
Ghost characters, character varieties and abelian knot contact homology

10:05 – 10:30 Shinnosuke Suzuki (Meijo University)
On SL(2,C)-representations of the fundamental group of the 2-fold branched cover of 3-sphere branched along a knot

10:45 – 11:30 Shinichi Tajima (University of Tukuba)
Limiting tangent spaces and local cohomology

13:00 – 13:45 Alden Walker (Center for Communication Research)
Surface subgroups from linear programming

14:00 – 14:15 Eri Kamikawa (Meiji University)
A recognition system of knot diagram image

14:20 – 14:35 Yuumu Rikiishi (Meiji University)
BeadsKnot - A knot drawing system which allows us to simulate Reidemister Moves

14:40 – 14:55 Kento Nakamura (Meiji University)
An interactive visualization system on a family of Kleinian groups based on Schottky groups

15:10 – 15:55 Koji Nuida (National Institute of Advanced Industrial Science and Technology)
How to apply topology to cryptography, hopefully

16:10 – 16:40 Takuya Sakasai (University of Tokyo)
An abelian quotient of the symplectic derivation Lie algebra of the free Lie algebra

16:50 – 17:35 Haruko Takayama (Nishi) (Josai University)
Polyhedral structures of the configuration space of points on $P^1$

**October 30**

9:30 – 10:00 Ayaka Shimizu (National Institute of Technology, Gunma College)
Reductivity problem on knot projections

10:10 – 10:55 Alden Walker (Center for Communication Research)
Rigorous mathematical certificates

11:10 – 11:40 Shun Wakatsuki (University of Tokyo)
Computation of string operations using rational homotopy theory

11:50 – 12:35 Masaaki Wada (Osaka University)
OPTi 4

Organizers: Kazuhiro Ichihara (Nihon University), Yasuharu Nakae (Akita University)

# Abstracts

**October 28**

13:30 – 14:15 Yuichi Kabaya (Kitami Institute of Technology)
Optimal Kazhdan constants and semidefinite programming

> Narutaka Ozawa found a new necessary and sufficient criterion for Kazhdan's property (T) for finitely generated groups, which gives an explicit lower bound of the spectral gap, and the Kazhdan constant. Using his criterion with semidefinite programming, Netzer-Thom gave a lower bound of the Kazhdan constant for $SL_3(\mathbb{Z})$ with respect to the generating set given by elementary matrices. This improved the previous results by Shalom, Kassabov. I will show more examples and computations.

14:30 – 15:15 Genki Kusano (Tohoku University)
Statistical analysis of persistent homology

> In topological data analysis(TDA), a persistent homology is widely used to describe the robust and noisy topological properties in data, and it has a compact form called a persistence diagram. In this talk, I will propose a kernel method for persistence diagrams to develop a statistical framework in TDA. Then, a new distance on persistence diagrams is defined by our kernel, and satisfies the stability property. As an application, we show that our method can clearly detect the glass transition temperature in SiO2 from their geometric structures.

15:30 – 16:15 Hiroki Kodama (University of Tokyo)
Protein structure analysis and $SO(3)$

> Proteins are large molecules, consisting of chains of 20 types of amino acid residues. This long chain, polypeptide, forms specific three-dimensional structure by means of hydrogen bond.
>
> In this talk we present a modeling of protein three-dimensional structure that uses the rotation group $SO(3)$ to represent the relative direction of connecting peptide units. We will show some application on acutual proteins.
>
> We also refer to the application for protein folding problem, by J. E. Andersen et al.

16:30 – 17:15 Szilárd Zsolt Fazekas (Akita University)
Regular languages closed under word operations

> The central topic of the talk is the extension of iterated word operations to all words of a given language. Several word operations have been considered in this context, such as duplication, power operation, pseudopalindromic completion and iterated hairpin completion. For some of these operations the result of applying them a regular language is not necessarily regular. The decidability of whether the resulting language is regular is a hard problem in most cases. We present a survey of the cases when the result is always regular as well as characterization and related decidability theorems in the other cases.

17:30 – 18:00 Masaaki Suzuki (Meiji University)
Epimorphisms between two-bridge knot groups and their crossing numbers

In this talk, we consider the relationship between epimorphisms of two-bridge knot groups and their crossing numbers. In particular, if there exists an epimorphism from the knot group of a two-bridge knot $K$ onto that of another knot $K'$, then the crossing number of $K$ is greater than or equal to three times of that of $K'$. Moreover, we formulate the generating function which determines the number of two-bridge knot groups admitting epimorphisms onto the knot group of a given two-bridge knot.

**October 29**

9:30 – 10:00 Fumikazu Nagasato (Meijo university)
Ghost characters, character varieties and abelian knot contact homology

We introduce a ghost character of a knot $K$ using the trace-free characters $S_0(K)$ of SL(2,C)-representations of the knot group $G(K)$. This gives a tool to describe exactly the relationship of degree 0 abelian knot contact homology $HC_0^{ab}(K)$ with the character variety $X(\Sigma_2 K)$ of the 2-fold branched cover $\Sigma_2 K$ of 3-sphere branched along $K$. Using this, we give some criteria to check
(1) whether Ng's conjecture concerned with the above relationship holds true or not,
(2) when the map $\widehat{\Phi} : S_0(K) \to X(\Sigma_2 K)$ constructed in [F. Nagasato-Y. Yamaguchi: On the geometry of the slice of trace-free $SL_2(\mathbb{C})$-characters of a knot group, Math. Ann. **354** (2012), 967–1002] is not surjective.

10:05 – 10:30 Shinnosuke Suzuki (Meijo University)
On SL(2,C)-representations of the fundamental group of the 2-fold branched cover of 3-sphere branched along a knot

We will calculate concretely the fundamental group of the 2-fold branched cover of 3-sphere branched along a certain knot. Then we will discuss a relationship of its $SL_2(\mathbb{C})$-representations and ghost characters. Using this, we will give a knot $K$ such that the map $\widehat{\Phi} : S_0(K) \to X(\Sigma_2 K)$ given in [F. Nagasato-Y. Yamaguchi: On the geometry of the slice of trace-free $SL_2(\mathbb{C})$-characters of a knot group, Math. Ann. **354** (2012), 967–1002] is not surjective.

10:45 – 11:30 Shinichi Tajima (University of Tukuba)
Limiting tangent spaces and local cohomology

The concept of a llimiting tangent space was introduced by H. Whitney in his study of stratifications. Limiting tangent spaces have been extensively utilized in various ways in singularity theory, especially in problems that involve Whitney stratifications.

In this talk, limiting tangent hyperplanes associated with hypersurface isolated singularities are considered in the context of symbolic computation. A new effective method is proposed to compute the limiting tangent space of a given hypersurface. The key of the proposed method is the concept of parametric local cohomology systems. Proposed method can provide the decomposition of the limiting tangent space by Milnor numbers of hyperplane sections of a given hypersurface. The resulting algorithm has been implemented in the computer algebra system Risa/Asir.

13:00 – 13:45 Alden Walker (Center for Communication Research)
Surface subgroups from linear programming

Gromov asked whether every one-ended hyperbolic group contains a subgroup isomorphic to the fundamental group of a closed hyperbolic surface. The question is still open in general, but the answer is known to be "yes" in many infinite families of groups. In this talk, I'll give the proof for random HNN extensions of free groups. In this case, we describe a certificate which guarantees the existence of a surface subgroup, and we show how to produce this certificate in a random HNN extension. We also use the certificate computationally to prove the existence of a surface subgroup in a specific HNN extension which was conjectured not to contain such a subgroup. This is joint with Danny Calegari.

14:00 – 14:15 Eri Kamikawa (Meiji University)
A recognition system of knot diagram image

For beginners of knot theory, it is difficult to copy a knot diagram on textbooks and papers onto their notebooks. It is also confusing for anyone to modify knot diagrams on their notebooks. In this study, we propose a system which captures an image and a photo of a knot diagram and recognizes curves and crossings from the image to extract the data of the knot diagram. First, this system divides the image into small squares and recognizes a line segment which is appeared in each square to get the graph structure. Next, it removes thorn shaped segments and add a segment at a gap and recognizes the shapes of crossings. Here, this procedure might fail because of the size of division, the system estimates the thickness of lines from the original image and tries the process several times by changing the size of squares. Thus it modifies the obtained picture of the knot with double spring scheme devised by Mr. Rikiishi. In the future, we develop a system which allows learners of knot theory to transform and manipulate knot diagrams on the screen over our system.

14:20 – 14:35 Yuumu Rikiishi (Meiji University)
BeadsKnot - A knot drawing system which allows us to simulate Reidemister Moves

In ' topology and computer 2015 ', the speaker presented on a system BeadsKnot, which allows users to edit knot diagrams on the screen and to apply smoothing operation. In this system, a knot diagram is represented by beads and segments. In this talk we introduce a new version of BeadsKnot that was added three types of Reidemeister moves. Users are allowed to operate Reidemeister moves by a mouse dragging. The system can determine the type of Reidemeister move only from the mouse cursor track.

14:40 – 14:55 Kento Nakamura (Meiji University)
An interactive visualization system on a family of Kleinian groups based on Schottky groups

We developed an algorithm for drawing images of a family of Kleinian groups based on Schottky groups. This algorithm is called an Iterated Inversion System (IIS). IIS allows us to perform calculation in parallel and render images fast. The speaker is developing an interactive visualization system using IIS. In this system, any generator is given by the inversion of a circle (or a sphere or a plane) or a composition of some inversions. Here it is important for us to observe and control these generators in a geometrical way. We can easily explore complicated groups including 4 dimensional Kleinian groups. This system will be helpful to researchers and also fractal artists. In this talk, we introduce the system and how to construct generators and apply it to IIS.

15:10 – 15:55 Koji Nuida (National Institute of Advanced Industrial Science and Technology)
How to apply topology to cryptography, hopefully

Recent studies of cryptography have interesting connections to many subjects of mathematics. In this talk, the speaker will introduce parts of such relationship between mathematics and cryptography, and would like to tell "dreams" about possible applications of topology to cryptography.

16:10 – 16:40 Takuya Sakasai (University of Tokyo)
An abelian quotient of the symplectic derivation Lie algebra of the free Lie algebra

We construct an abelian quotient of the symplectic derivation Lie algebra of the free Lie algebra generated by the fundamental representation of the symplectic group. More specifically, we show that the weight 12 part of the abelianization of the Lie algebra is stably 1-dimensional. The computation is done with the aid of computers. This is a joint work with Shigeyuki Morita and Masaaki Suzuki.

16:50 – 17:35 Haruko Takayama (Nishi) (Josai University)
Polyhedral structures of the configuration space of points on $P^1$

Thurston showed that the moduli space of configurations of points on the complex projective line admits a family of hyperbolic metrics parameterized by sequences of real numbers $(a_1, \cdots, a_n)$ with $0 < a_i < 1$ summing up to two. Their metric completions are $(n-3)$-dimensional complex hyperbolic cone manifolds. In this talk, I will discuss the real part of these cone manifolds by relating them to the moduli space of Euclidean polygons and present their polyhedral structures. Moreover, I will present a GAP software tool which calculates their fundamental groups, homology groups, as well as further combinatorial data. This is joint work with J. Spreer.

**October 30**

9:30 – 10:00 Ayaka Shimizu (National Institute of Technology, Gunma College)
Reductivity problem on knot projections

A knot projection on a 2-sphere is a regular immersion of a circle to the sphere. We say that a knot projection is reducible if we can draw a circle on the sphere which intersects the knot projection transversely at one crossing. Otherwise, we say that it is reduced. We consider a local move, called an inverse-half-twisted splice, which is a splice at a crossing such that the orientation of the knot projection is not preserved. We always obtain a knot projection (not a link projection) from a knot projection by this move. The reductivity of a knot projection is the minimum number of inverse-half-twisted splices needed to obtain a reducible knot projection from the knot projection. It is shown that the reductivity is four or less for any knot projection. However, it is unknown if there exists a knot projection with reductivity four. In this talk, we give an unavoidable set of parts for a knot projection with reductivity four (if exists) by considering 4-gons. This is a joint work with Yui Onoda.

10:10 – 10:55 Alden Walker (Center for Communication Research)
Rigorous mathematical certificates

I'll describe how we can actually compute the subgroup certificate from my "surface subgroups" talk and how this program fits into a much larger program designed to construct fatgraphs. I'll discuss another situation involving iterated function systems in which we can produce a certificate that a certain fractal is connected, allowing us to prove the existence of geometrically nontrivial components of Schottky space. If time permits, I'll give some more examples of rigorously experiments in pure math. Could involve work joint with Juliette Bavard, Danny Calegari, and Sarah Koch.

11:10 – 11:40 Shun Wakatsuki (University of Tokyo)
Computation of string operations using rational homotopy theory

Let $LM$ be the free loop space of a rational Gorenstein space $M$. Félix and Thomas defined string operations on the homology of $LM$. Using rational homotopy theory, we have an elementary description of these operations. In this talk, we discuss computations of string operations by this description.

11:50 – 12:35 Masaaki Wada (Osaka University)
OPTi 4

OPTi is a program for visualizing quasi-conformal deformations of the once-punctured torus groups. I started working on the program in around 1996, so it is a 20-year-old project now. OPTi has been inspiring many researchers in hyperbolic geometry and Kleinian groups.

The first versions of OPTi ran on Macs with 680x0 cpu's and PowerPC. In 2005 Apple started transition from PowerPC to Intel processors, and in around 2009 they dropped support for PowerPC. The last OS version on which OPTi 3 runs was 10.6 for a long time. I knew mathematicians who kept their Mac from upgrading to newer OS just to run OPTi. After several years of preparation, I finally released OPTi 4 this May. I would like to talk about what it took me to update OPTi for the newest Mac OS X.