Regular languages closed under word operations

Szilárd Zsolt Fazekas

Akita University

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- $w^0 = \lambda$ and $w^{n+1} = w^n w$, $\forall n \geq 0$: powers of a word $w \in \Sigma^*$
- $w^* = \{w^0, w^1, w^2, \dots\}$.

A finite automaton is a quintuple $\mathcal{A} = \langle \Sigma, Q, q_0, F, \sigma \} \rangle$ where

- Σ is the *input alphabet*,
- Q is a finite set called the *set of states*,
- $q_0 \in Q$ is the *initial state*,
- $F \subseteq Q$ is the *set of final states* and
- $\sigma : Q \times \Sigma \to 2^Q$ is the *transition function*.

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If $\forall q \in Q, a \in \Sigma : |\sigma(q, a)| \leq 1$ then $\mathcal{A}$ is deterministic, otherwise nondeterministic.

$L(\mathcal{A})$, the language accepted by the finite automaton $\mathcal{A}$ is the set of all words $a_1 a_2 \ldots a_n$ $(a_i \in \Sigma)$, such that there exist states $p_0, \ldots, p_n$ such that

$$\forall i \in \{1, \ldots, n\} : p_i \in \sigma(p_{i-1}, a_i),$$

$$p_0 = q_0, p_n \in F$$

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A language is regular iff it is accepted by a finite automaton.

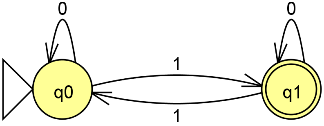
Example

$\mathcal{A} = \langle \{0,1\}, \{q_0, q_1\}, q_0, \{q_1\}, \sigma \rangle$, where the transition function σ is:

σ	0	1
q_0	q_0	q_1
q_1	q_1	q_0

$$L(\mathcal{A}) = \{w \in \Sigma^* \mid \exists k \geq 0 : \ |w|_1 = 2k+1\},$$

that is all binary words having an odd number of 1's.



Quasi order (preorder): reflexive and transitive binary relation

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Well quasi order: any infinite sequence of elements $x_0, x_1, \ldots$ contains an increasing pair $x_i \leq x_j$ with $i < j$. So:

- no infinite decreasing series
- no antichain (infinite series of pairwise incomparable elements)

Definition

For $u, v \in \Sigma^*$:

$u \leq v$: u is a subsequence (subword, scattered subword) of v if $u = x_1 \cdots x_n$ and $v = y_0 x_1 y_1 x_2 y_2 \cdots y_n$ for some $x_i, y_j \in \Sigma^*$.

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Definition

Words $w_1, w_2, \ldots, w_n$ form a basis of L if:

- $\forall v \in L, \forall i \in \{1, \ldots, n\} : v \leq w_i \Rightarrow v = w_i$ - they are all minimal in L and
- $\forall v \in L, \exists i \in \{1, \ldots, n\} : w_i \leq v$ - they generate L

Lemma (Higman, 1952)

The subsequence relation is a well-quasi-order.

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Theorem (Haines, 1969)

For an arbitrary language $L \subseteq A^*$ both sets

$$Down(L) = \{v \in A^* | \exists w \in L : v \leq w\}$$

$$Up(L) = \{v \in A^* | \exists w \in L : w \leq v\},$$

are regular.

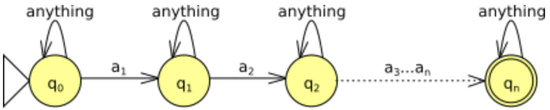

Figure: Automaton accepting all supersequences of a word $a_1 \cdots a_n$.

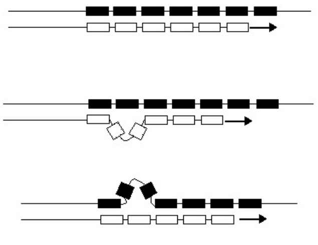

Figure: DNA replication with slippage leading to duplication

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- $\to^*$: the transitive closure of $\to$

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Closure under duplication:

- $u^{\to} = \{w \in \Sigma^* | u \to^* w\}$

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Closure under duplication:

- $u^{\rightarrow} = \{w \in \Sigma^* | u \rightarrow^* w\}$
- $L^{\rightarrow} = \bigcup_{u \in L} u^{\heartsuit}$

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- Bovet and Varricchio '92: copy languages are regular over a binary alphabet
- duplications considered again by Dassow, Mitrana, Păun in '99
- they show regularity of duplication closure of any binary word

Theorem (Bovet, Varricchio)

For an arbitrary language $L \subseteq \{a, b\}^*$, the language $L^{\to}$ is regular.

The argument: first show that **duplication over a binary alphabet is a well-quasi order on words**,

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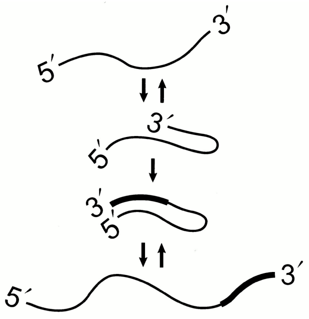
For an arbitrary language $L \subseteq \{a, b\}^*$, the language $L^{\rightarrow}$ is regular.

The argument: first show that **duplication over a binary alphabet is a well-quasi order on words**, then use the generalization of the Myhill-Nerode theorem:

Theorem (Ehrenfeucht, Haussler, Rozenberg)

A language L of a finitely generated free monoid is regular if and only if it is upwards closed with respect to a monotone well quasi order.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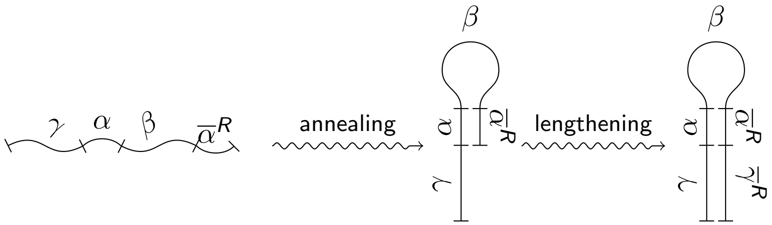

single strand hairpin hairpin completion

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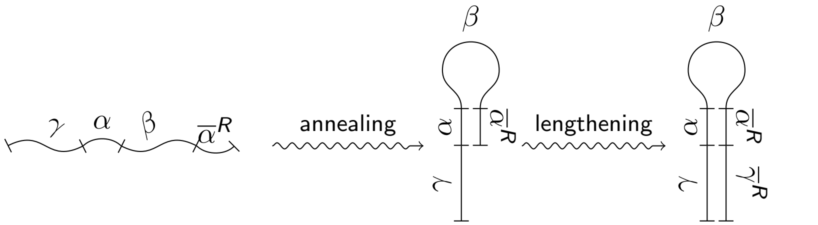

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hairpin

hairpin completion

Mathematical hairpin concept (Păun et al., 1991): a word in which some suffix is the mirrored complement of a non-overlapping factor.

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Noteworthy exception: "given a word[1], can we decide whether the iterated application of the operation leads to a regular language?"

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Approach it by a simpler operation, pseudopalindromic completion (F, Manea, Mercas, Shiskishima-Tsuji, 2014)

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palindrome: $w = w_1 \ldots w_n = w_n \ldots w_1 = w^R$

For $\theta : \Sigma^* \to \Sigma^*$, we say that θ is:
involution: $\theta^2(w) = w$

antimorphism: $\theta(w) = \theta(w_n) \cdots \theta(w_1)$

pseudopalindrome: $w = \theta(w_1 \ldots w_n) = \theta(w_n) \ldots \theta(w_1) = w^R$

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$uv \ltimes_R uv\theta(u)$: right *(θ-)completion* (of uv) with $|v| \geq 2$ a (θ-)pseudopalindrome

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Definition

$L^{\ltimes} = \{w \mid \exists u \in L : u \ltimes^* w\}$.

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$$a\bar{a}a$$

- *Step 1:* $\{a\bar{a}a\bar{a},$

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$a\overline{a}a$

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- *So forth ...* $(\bar{a}a)^n \bigcup (a\bar{a})^n$ *for* $n \geq 2$

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$$\bar{a}a\bar{a}aa\bar{a}$$

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Along the lines of the characterization of palindromic languages by [Horváth, Karhumäki, Kleijn 1987], we can characterize pseudopalindromic ones:

Theorem

A regular language $L \subseteq \Sigma^*$ is pseudopalindromic, iff it is a union of finitely many languages of the form $L_p = \{p\}$ or $L_{r,s,q} = qr(sr)^*q^R$ where p, r and s are pseudopalindromes, and q is an arbitrary word.

Theorem

$w^{\ltimes}$ is regular iff w has at most one pseudopalindromic prefix or one suffix, or for all words $w' \in w^{\ltimes_1}$ there exist unique pseudopalindromes p and q with $|p| \geq 2$, such that:

- $w' \in p(qp)^+$
- *w' has no pseudopalindromic prefixes except for the words in $p(qp)^*$.*

Theorem

For a regular language L, its iterated pseudopalindromic completion $L^{\ltimes}$ is regular if and only if L can be written as the union of disjoint regular languages L', L'', and L''', where

- $L'' = \{w \in L \mid w^{\ltimes \leq 1} = w^{\ltimes}\}$ *and the completion of every word in L'' is a subset of a finite union of languages of the form $up(qp)^*\overline{u}$, where $upqp\overline{u}$ has no pseudopalindromic prefixes and p, q are pseudopalindromes;*
- $L''' = \{w \in L \mid w^{\ltimes} \setminus (w^{\ltimes 1}) \neq \emptyset\}$ *and, for an integer $m \geq 0$ depending on L and pseudopalindromes p_i, q_i such that $p_i q_i$ have only one nontrivial prefix and only one nontrivial suffix, the completion of every word in L''' is a subset of $\bigcup_{i=1}^{m} p_i(q_i p_i)^+$;*
- $L' = L'^{\ltimes \leq 1} = L \setminus (L'' \cup L''')$.

Theorem

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Given a regular language L, it is decidable whether $L^{\ltimes}$ is regular. If the answer is YES, we can construct an automaton accepting $L^{\ltimes}$.

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The (primitive) root of a word $p \in \Sigma^+$ is the unique primitive word q such that $p = q^n$ for some $n \geq 1$. $\sqrt{p}$ denotes the root of p. For a language L, $\sqrt{L} = \{\sqrt{p} : p \in L\}$ is the root of L.

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$$\mathrm{pow}(L) = \{w^i \mid w \in L, i \geq 1\}.$$

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2009 F.: for any regular language YES (solution relies on unary case, but not on the HLL paper)

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Lemma

Let L be a regular language given by an NFA having n states. If $pow(L)$ is regular, then we have

$$pow(L) \subseteq L \cup \{\sqrt{u}^i \mid u \in L \wedge |u| \leq \max(n^2, m) \wedge i \geq 1\},$$

where m is the size of $\mathrm{Synt}(L)$.

Remark

Let L be a regular language given by an NFA having n states. If pow(L) is regular, then we have

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Lemma (Calbrix, Nivat)

Let L be a regular language of Σ^*. Then $\mathrm{pow}(L) = L$ if and only if there are regular languages $(L_i)_{1 \leq i \leq n}$ such that $L = \bigcup_{i=1}^{n} L_i^+$.

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$\downarrow$

For a regular language L it is decidable whether $pow(L) = L$.

Theorem

For a regular language L it is decidable whether $\mathrm{pow}(L)$ *is regular.*

Algorithm:

Input: an NFA $\mathcal{A} = \{\Sigma, Q, q_0, F, \sigma\}$.

Output: "YES", if $\mathrm{pow}(L(\mathcal{A}))$ is regular, and "NO" otherwise.

1. $U = \emptyset$ root of $\mathrm{pow}(L) \setminus L$

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2. FOR all words $w \in L(\mathcal{A})$ shorter than $2^{|Q|^2}$:
3. IF $w^* \setminus L(\mathcal{A}) \neq \emptyset$ THEN:

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2. FOR all words $w \in L(\mathcal{A})$ shorter than $2^{|Q|^2}$:
3. IF $w^* \setminus L(\mathcal{A}) \neq \emptyset$ THEN:
4. IF $\mathrm{pow}((\sqrt{w})^* \cap L(\mathcal{A}))$ is regular THEN add w to U
5. ELSE output "NO"

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2. FOR all words $w \in L(\mathcal{A})$ shorter than $2^{|Q|^2}$:
3. IF $w^* \setminus L(\mathcal{A}) \neq \emptyset$ THEN:
4. IF $\mathrm{pow}((\sqrt{w})^* \cap L(\mathcal{A}))$ is regular THEN add w to U
5. ELSE output "NO"
6. compute the syntactic monoid for $L' = L(\mathcal{A}) \setminus \bigcup_{u \in U} (\sqrt{u})^*$

Theorem

For a regular language L it is decidable whether $\mathrm{pow}(L)$ is regular.

Algorithm:
Input: an NFA $\mathcal{A} = \{\Sigma, Q, q_0, F, \sigma\}$.
Output: "YES", if $\mathrm{pow}(L(\mathcal{A}))$ is regular, and "NO" otherwise.

1. $U = \emptyset$ root of $\mathrm{pow}(L) \setminus L$
2. FOR all words $w \in L(\mathcal{A})$ shorter than $2^{|Q|^2}$:
3. IF $w^* \setminus L(\mathcal{A}) \neq \emptyset$ THEN:
4. IF $\mathrm{pow}((\sqrt{w})^* \cap L(\mathcal{A}))$ is regular THEN add w to U
5. ELSE output "NO"
6. compute the syntactic monoid for $L' = L(\mathcal{A}) \setminus \bigcup_{u \in U} (\sqrt{u})^*$
7. IF $L' = \mathrm{pow}(L')$ then output "YES"
8. ELSE output "NO"

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- Meaningful characterization of operations for which regularity of closure is decidable?

THANK YOU!