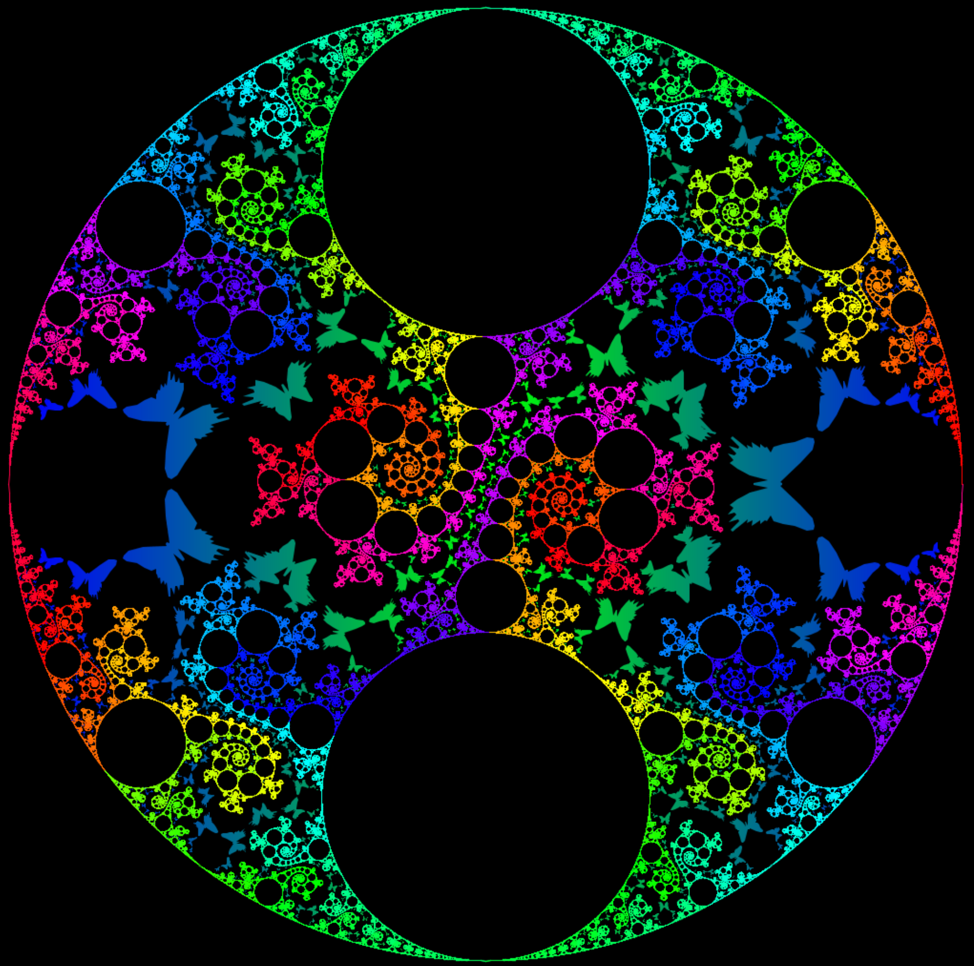
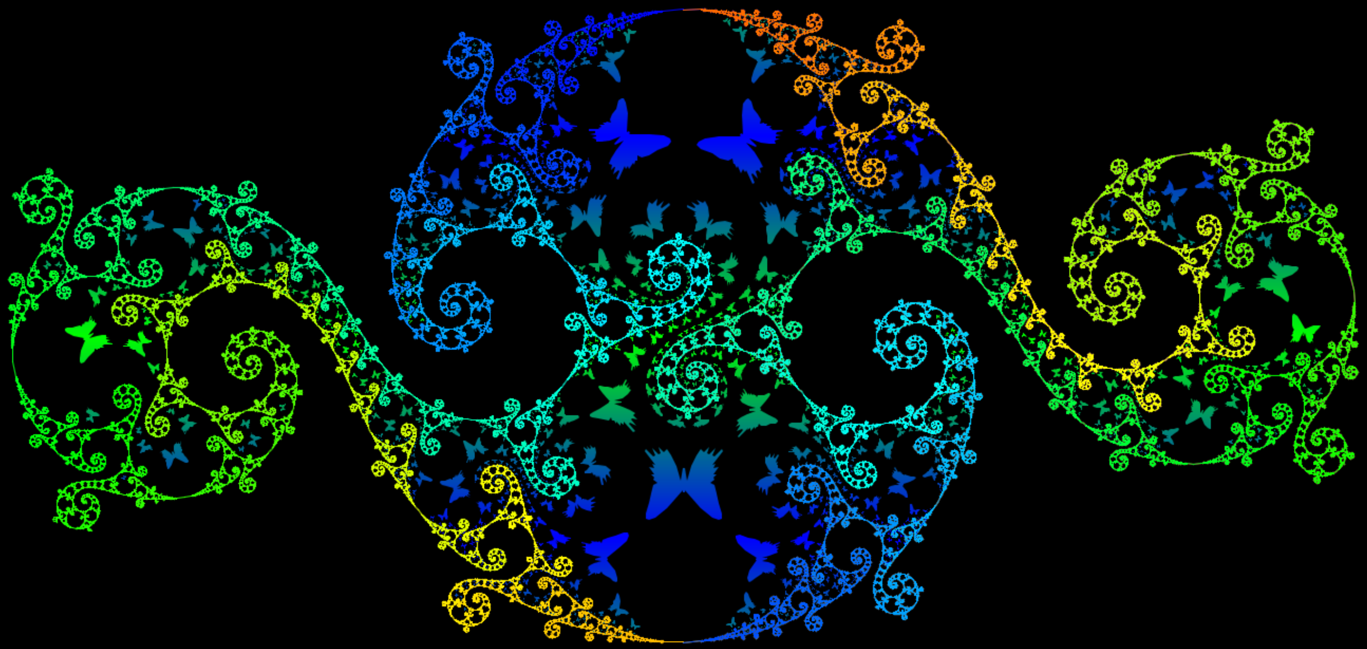
An interactive visualization system on a family of Kleinian groups based on Schottky groups

Meiji University
Kento Nakamura

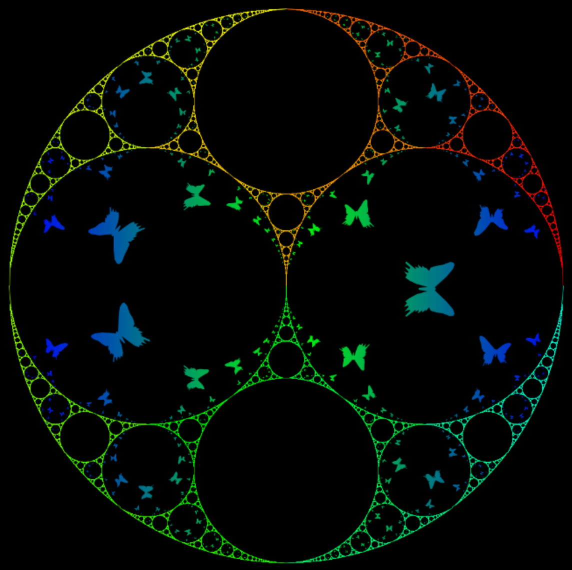
Visualization of Kleinian Groups



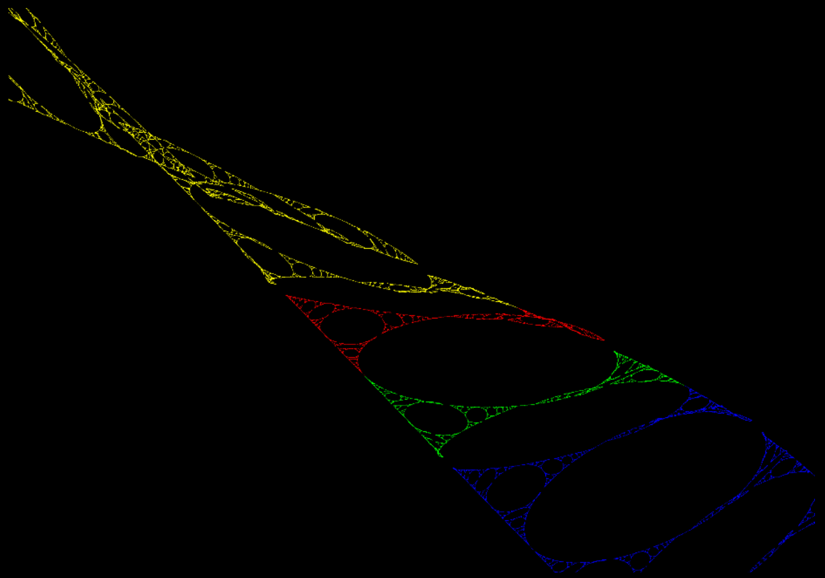


Generators of Kleinian Groups

- The generators are often given by algebraic expression

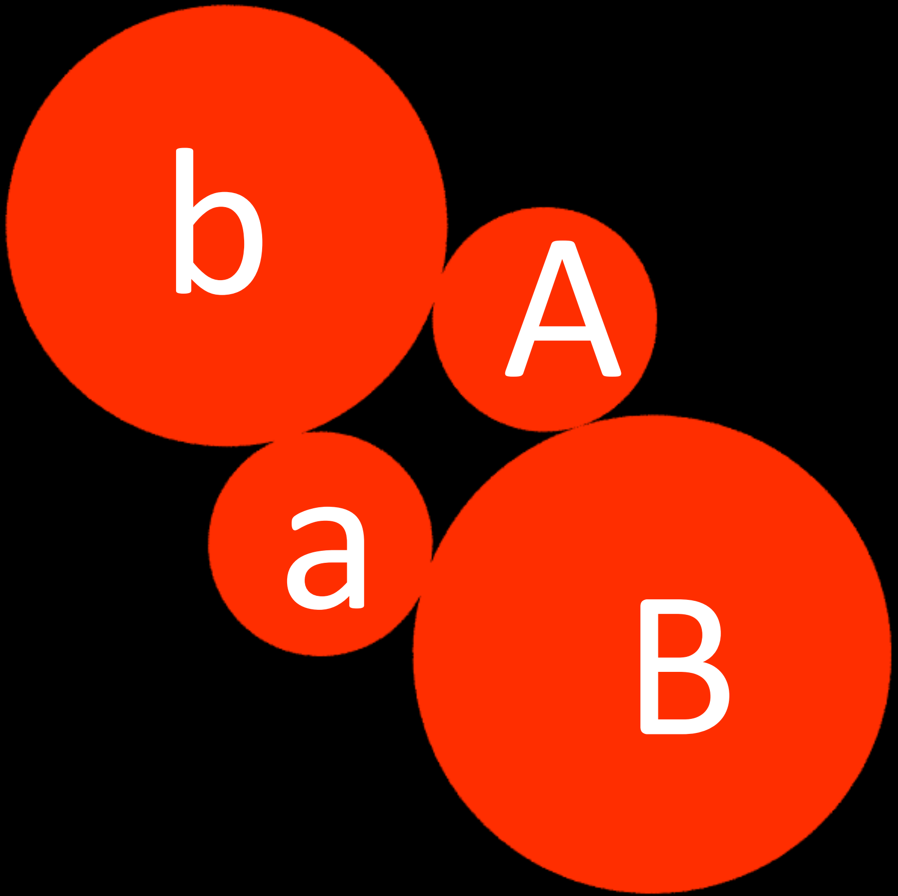


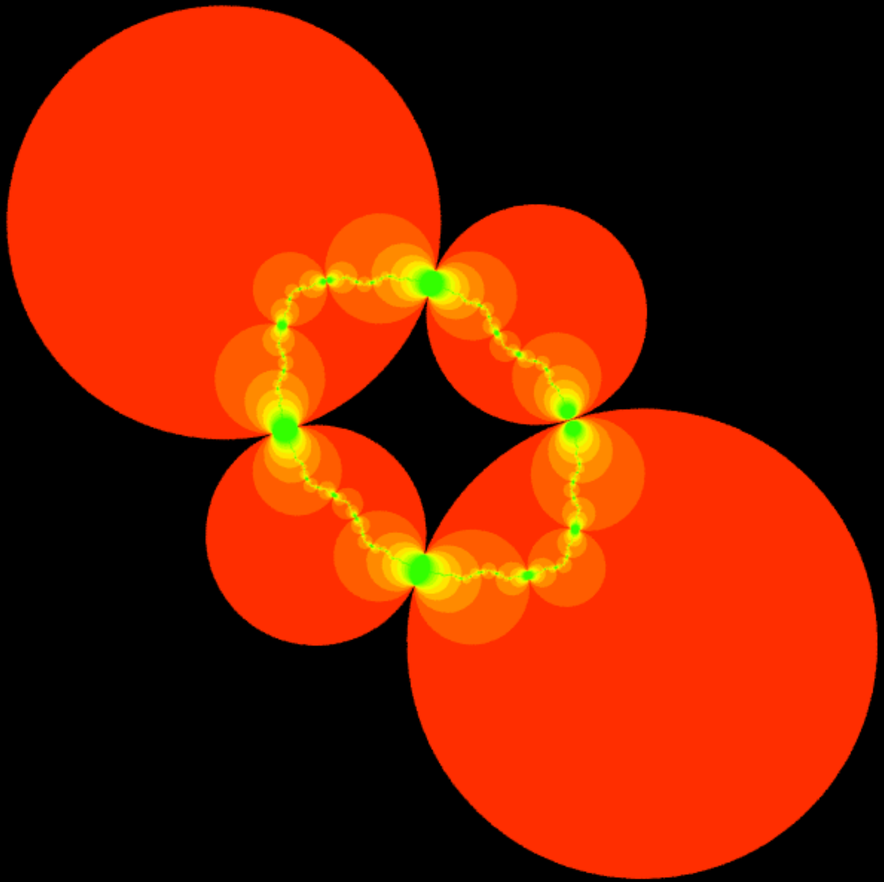
Grandma's Recipe
(from Indra's Pearls)



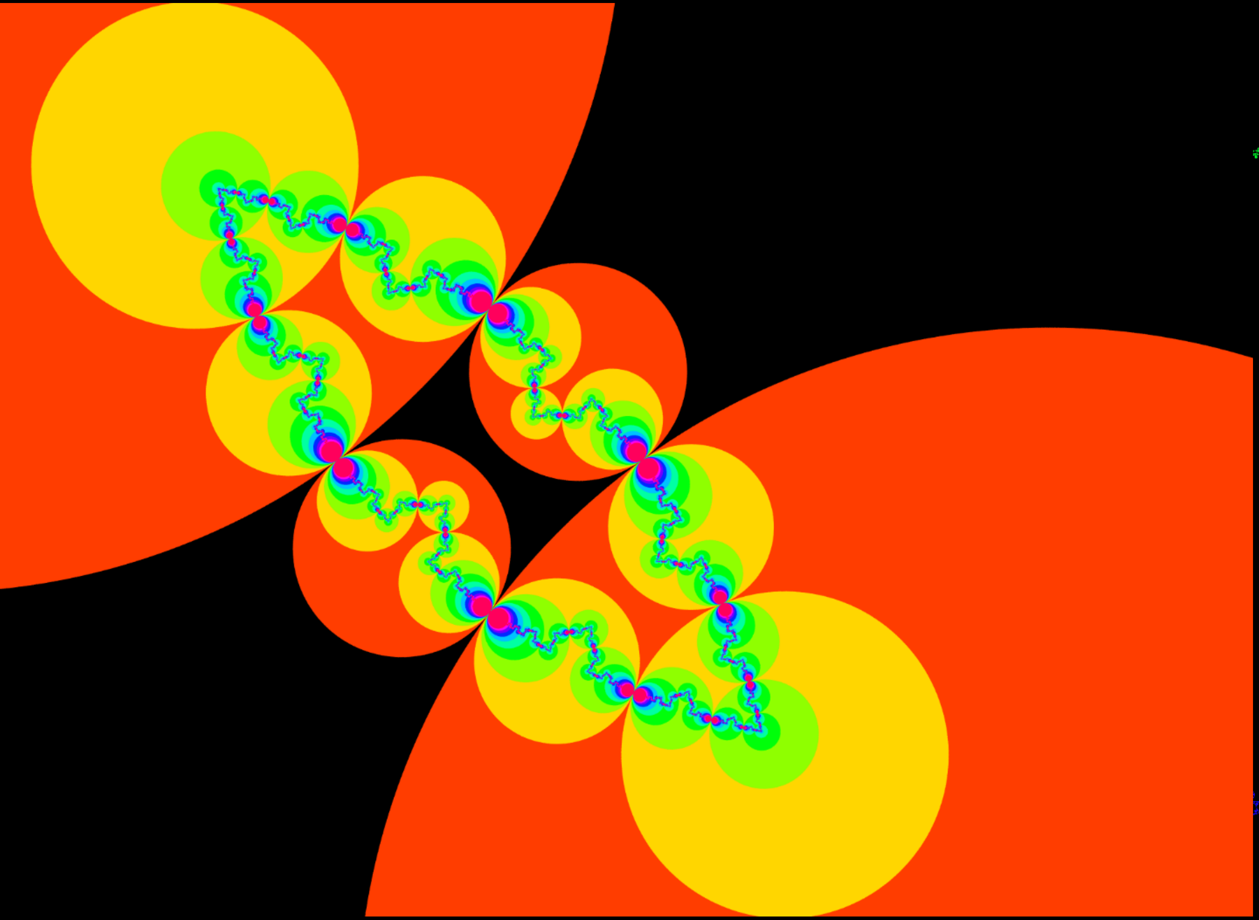
Compound Parabolic
(defined by Keita Sakugawa)

Schottky groups

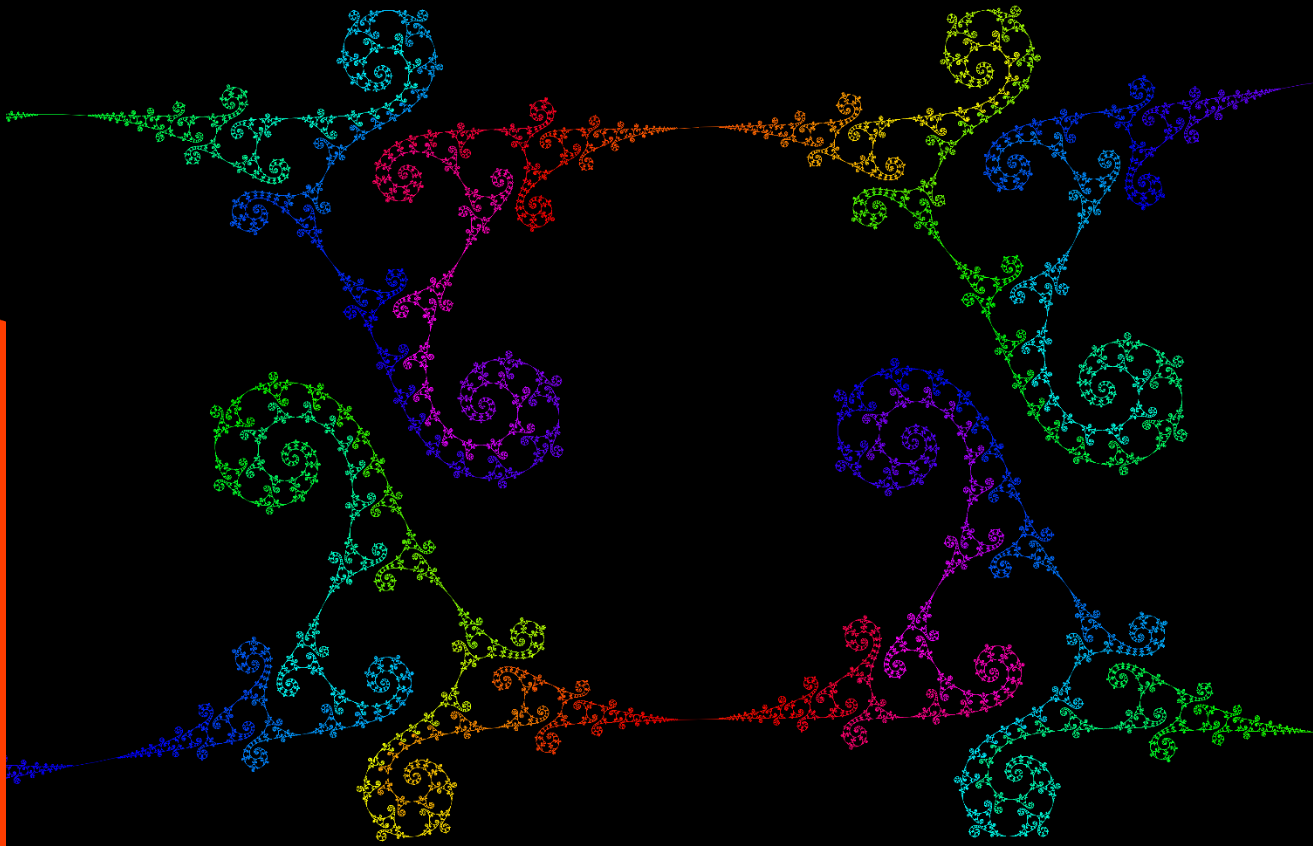




Visualization of Kleinian Groups



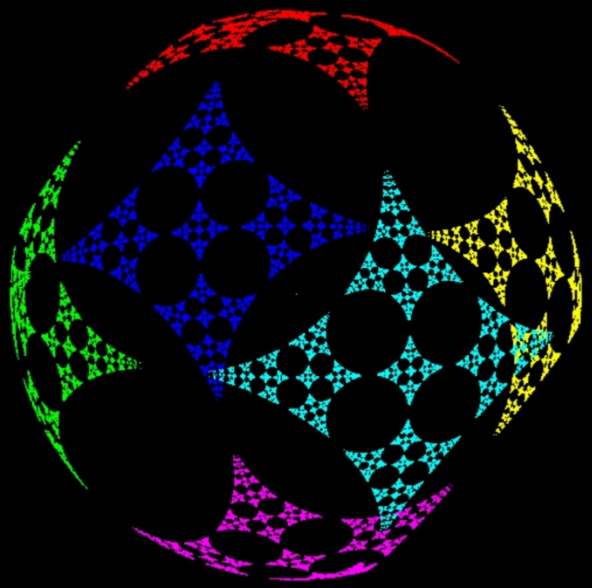
The orbit of Schottky circles



The limit set

Traditional ways

- Breadth First Search -> Orbit of transformation
- Depth First Search -> The Limit set



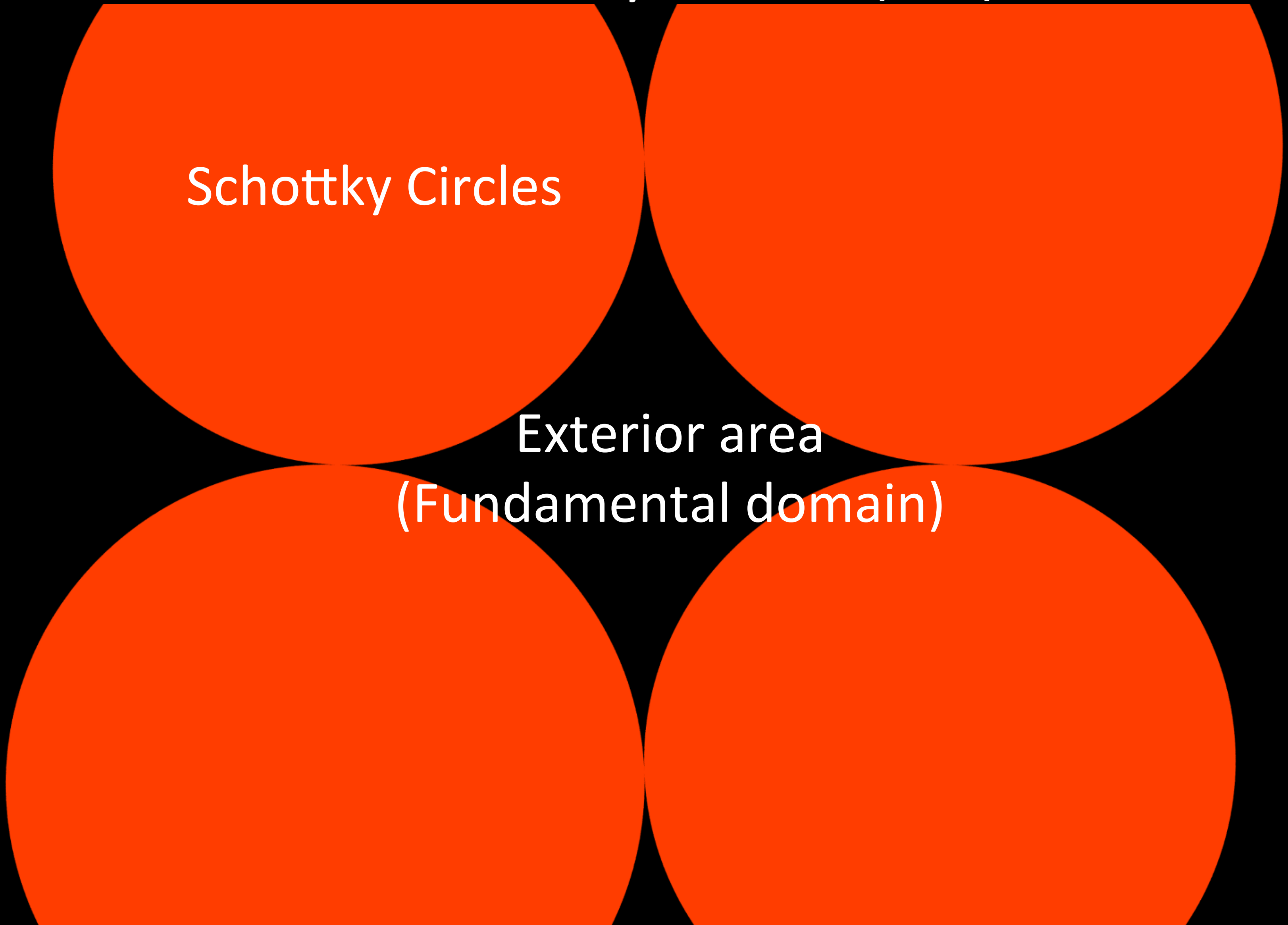
The Limit set

Faults of traversing Cayley graph

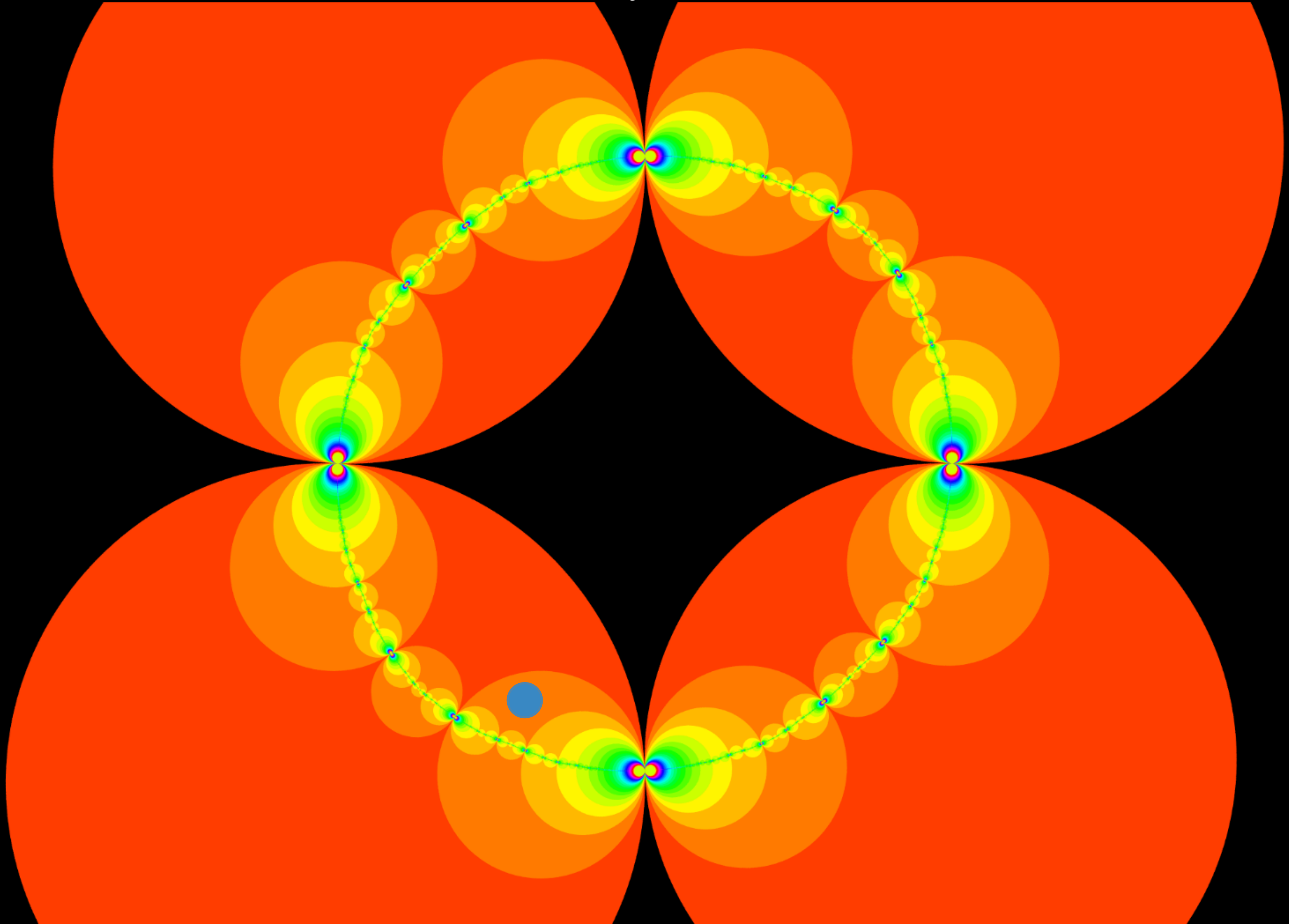
- If we increase Schottky circles, computational complexity increases exponentially.
- Traversing the Cayley graph is difficult to parallelize.
- It is difficult to draw a partial image of the limit set.

The Algorithm

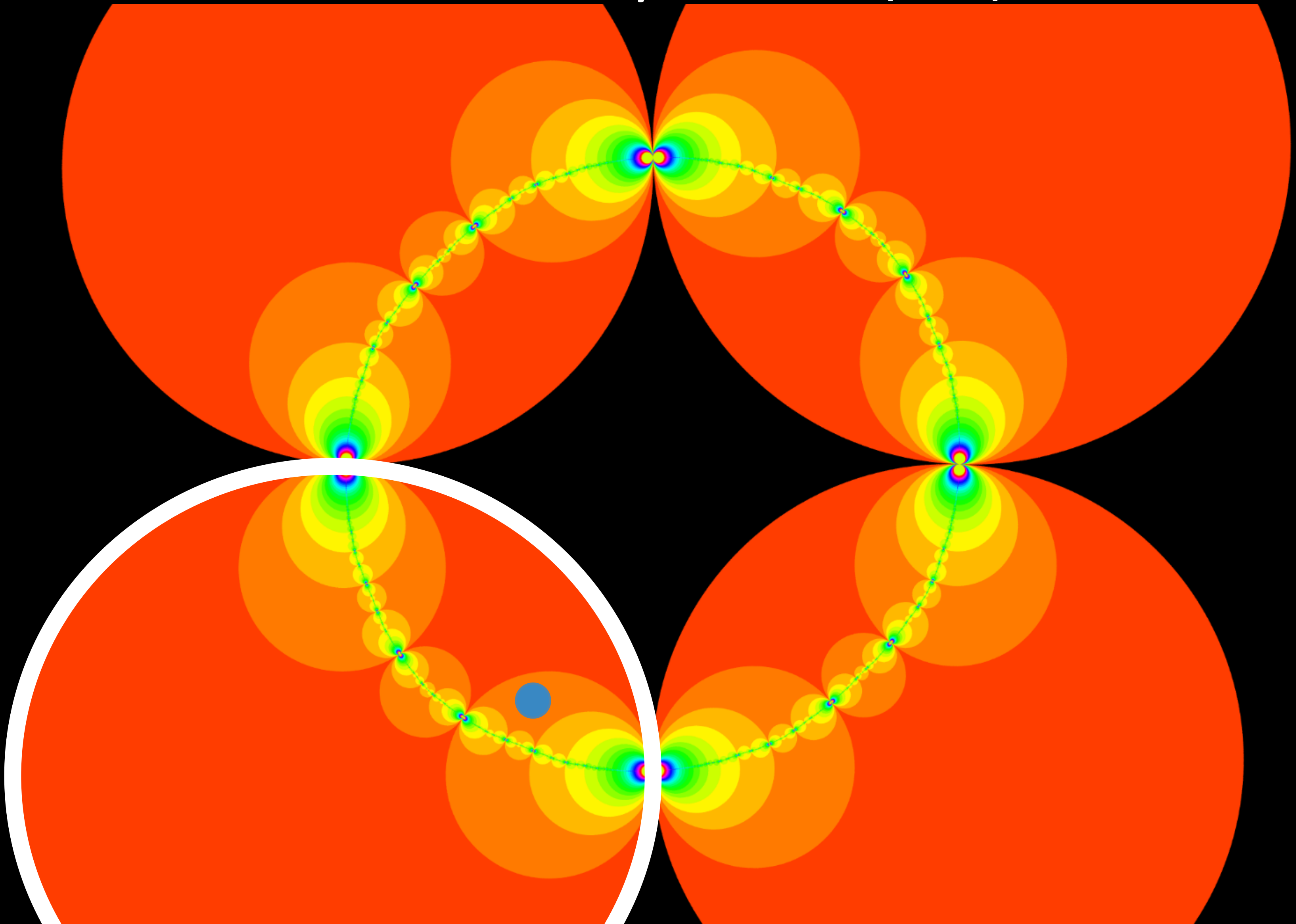
Iterated Inversion System (IIS)



Iterated Inversion System (IIS)



Iterated Inversion System (IIS)



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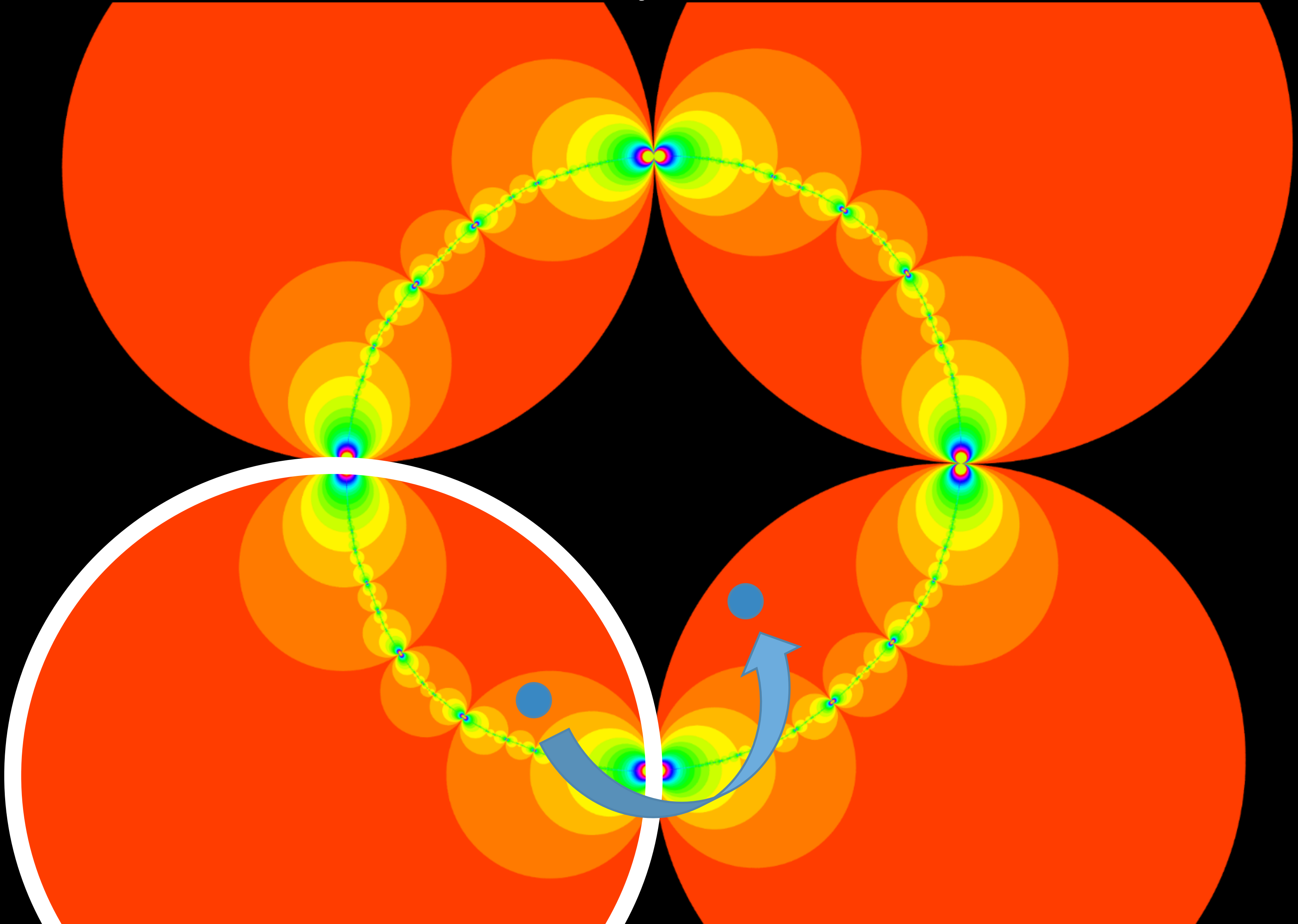
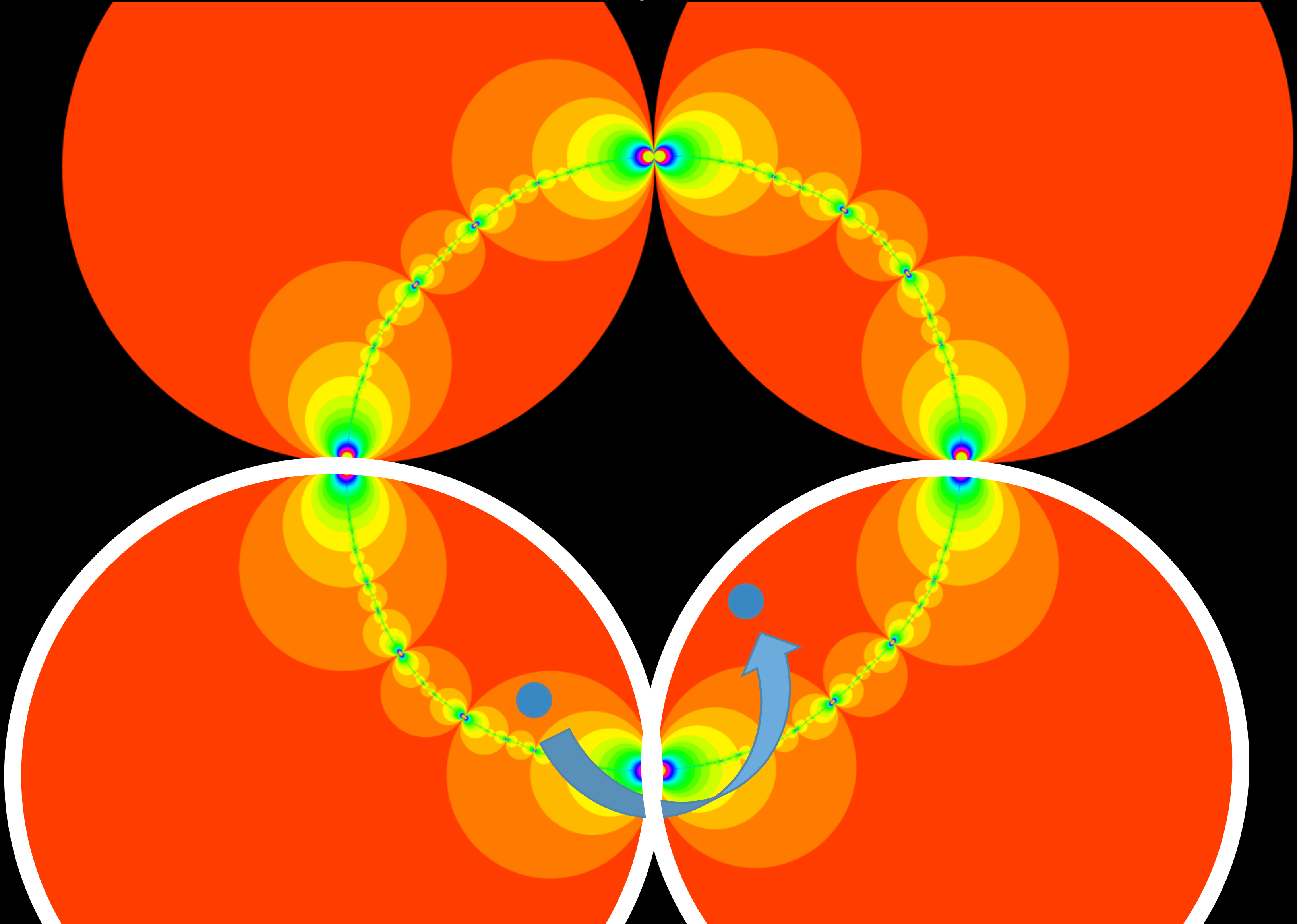


Iterated Inversion System (IIS)



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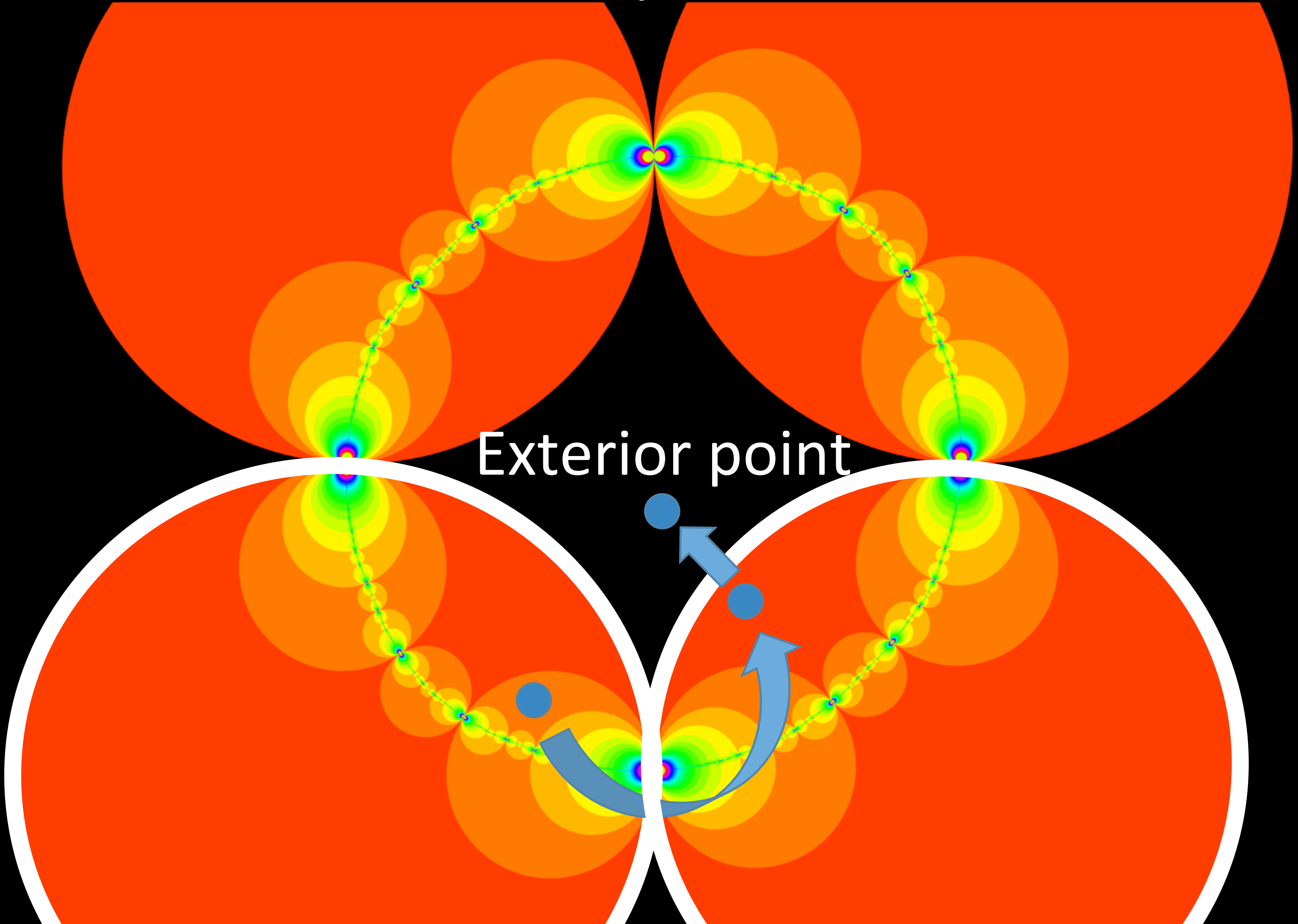
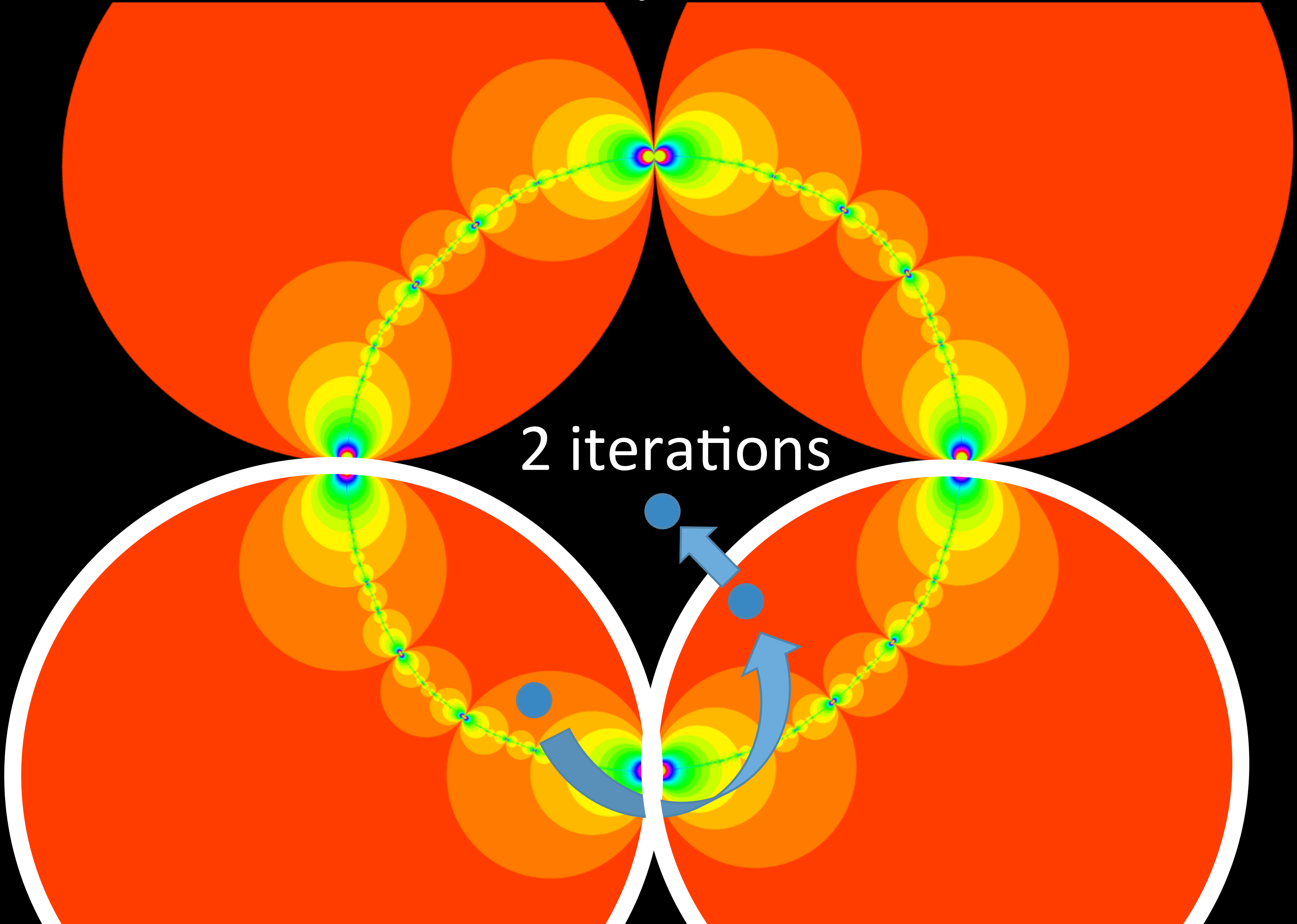
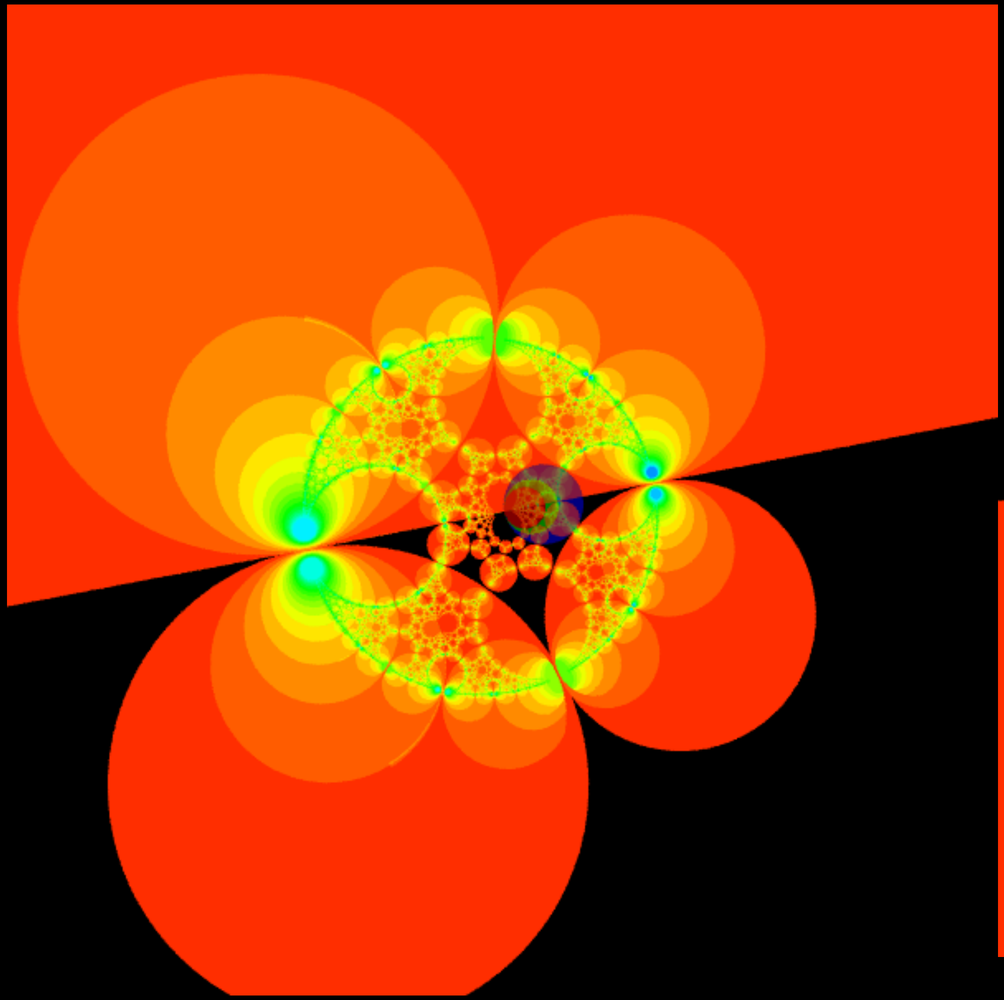
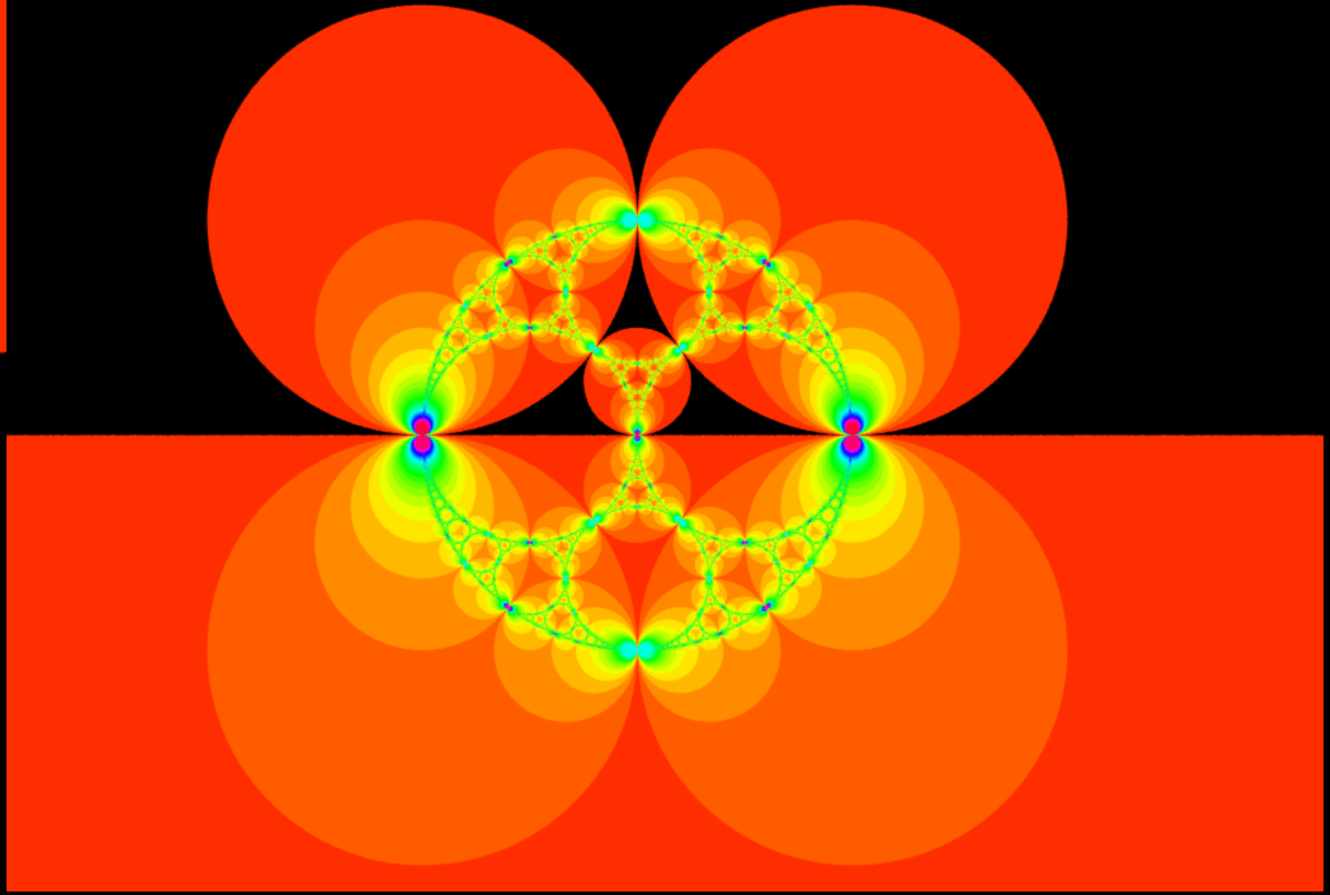


Iterated Inversion System (IIS)

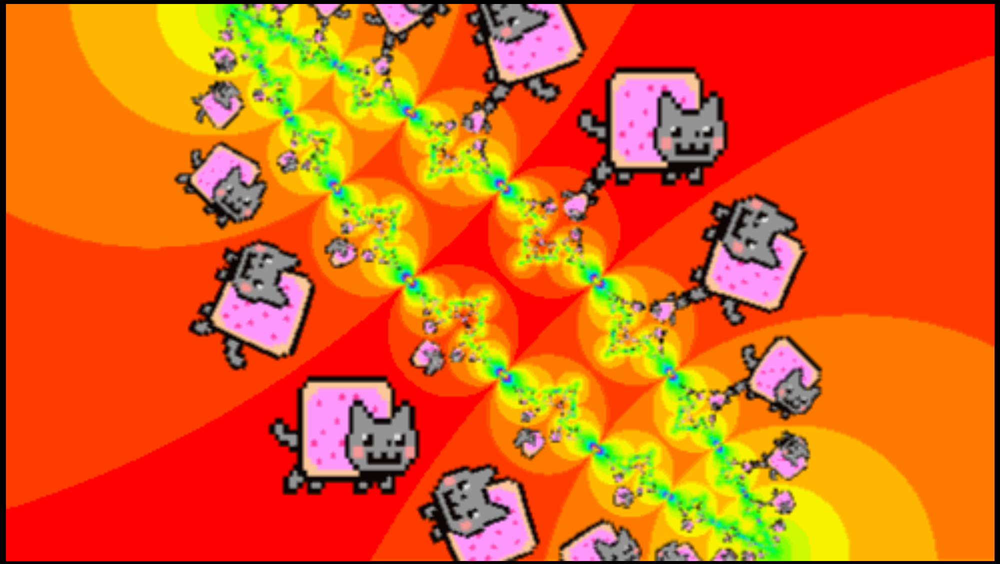


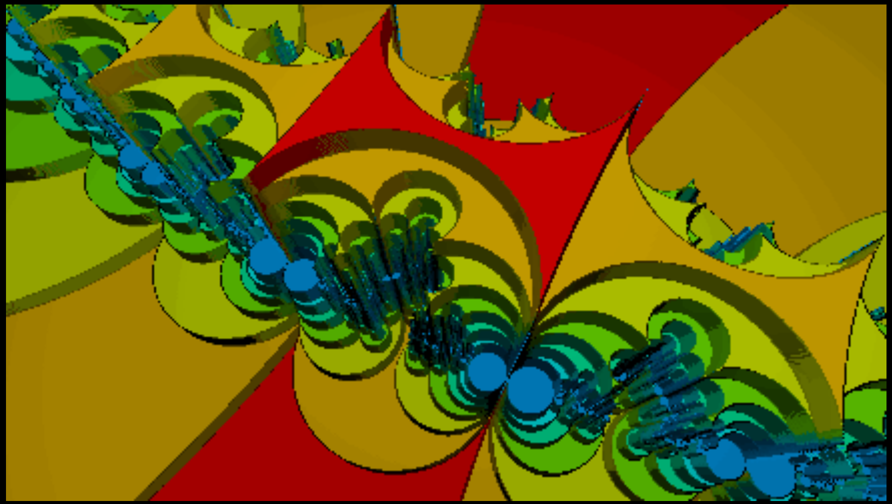
Rendered by Fragment Shader





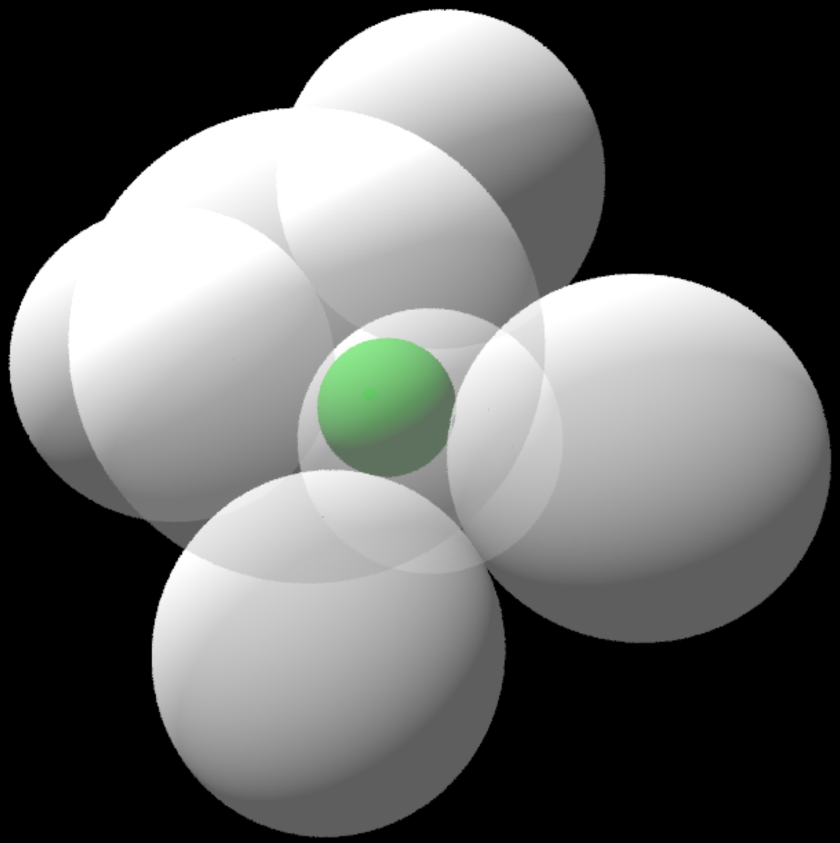
Applications
- Bitmap Orbit trap and Terrain raymarching

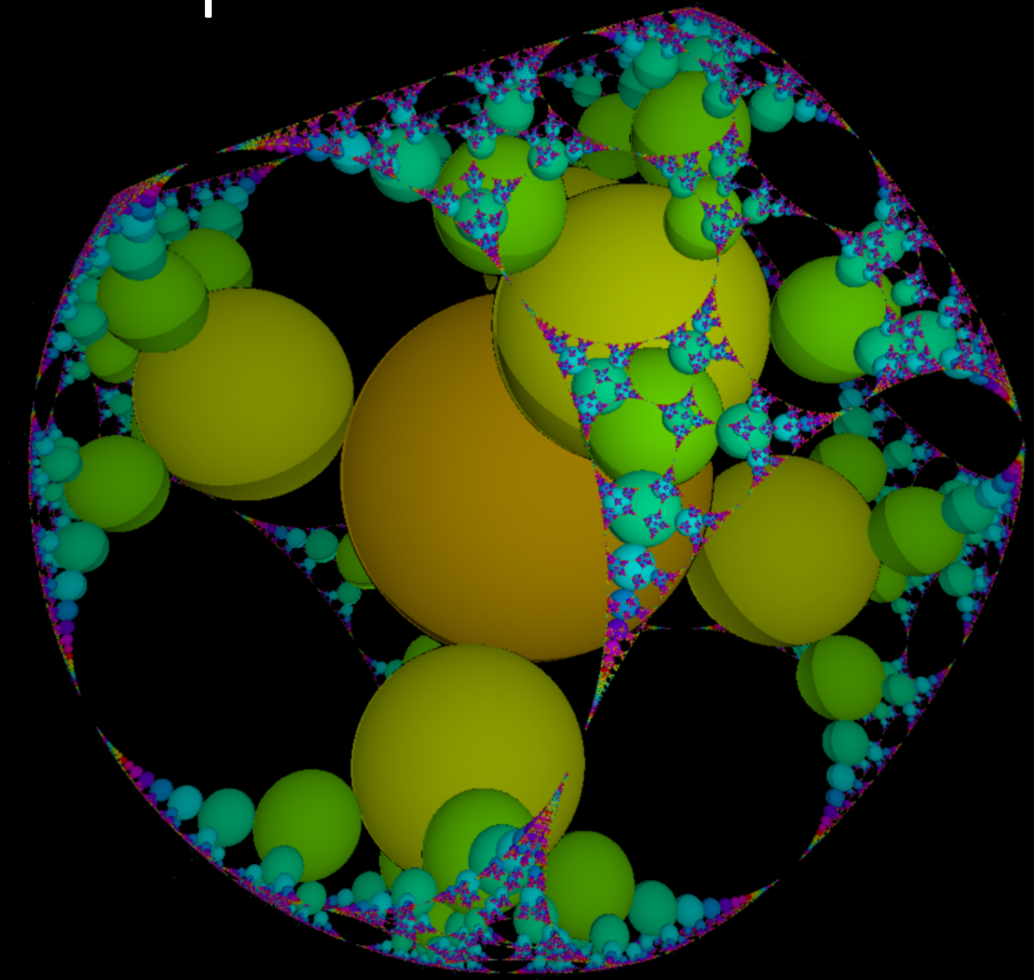


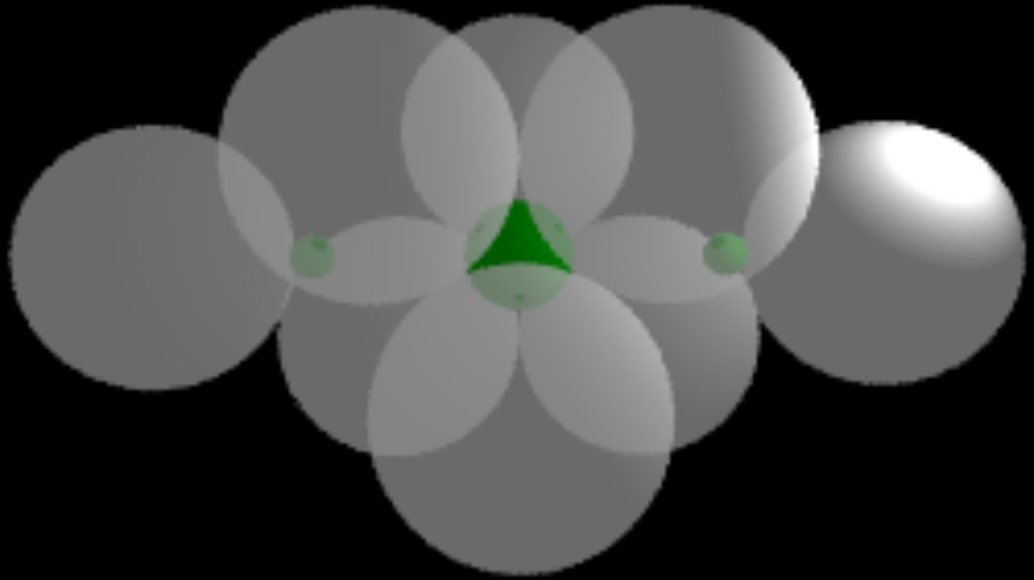


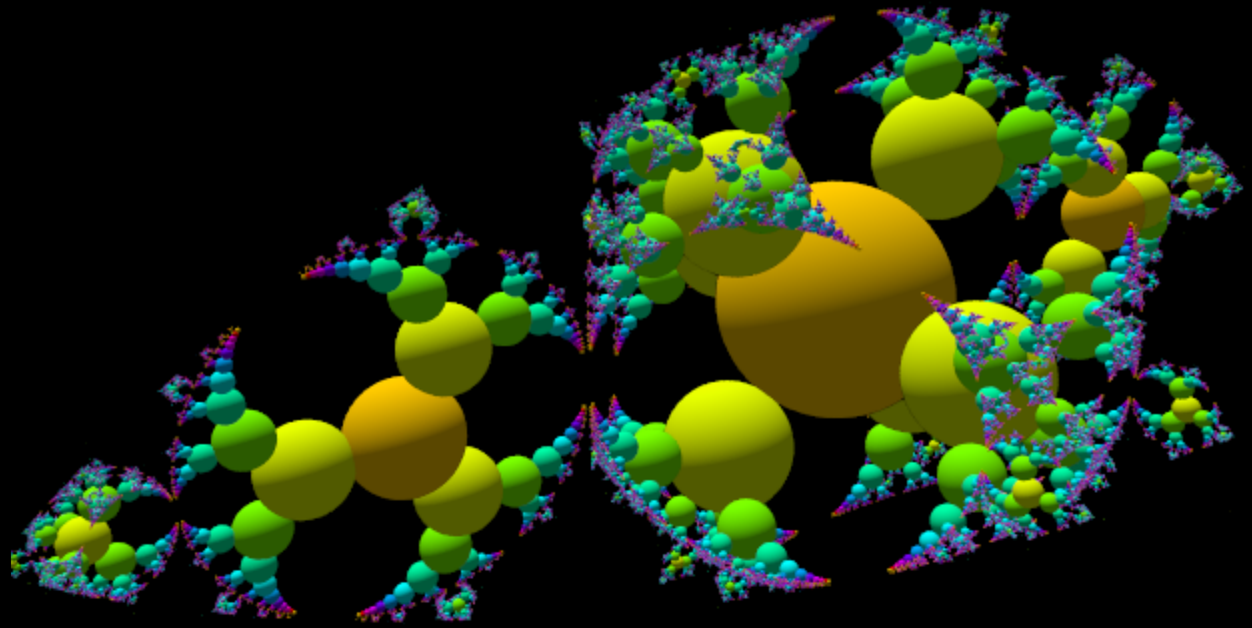
3D Extension

Schottky Spheres and Base sphere









A New Algorithm for rendering kissing Schottky groups,

Kento Nakamura, Kazushi Ahara

Bridges Finland 2016 Short papers

- Paper

  http://archive.bridgesmathart.org/2016/bridges2016-367.html

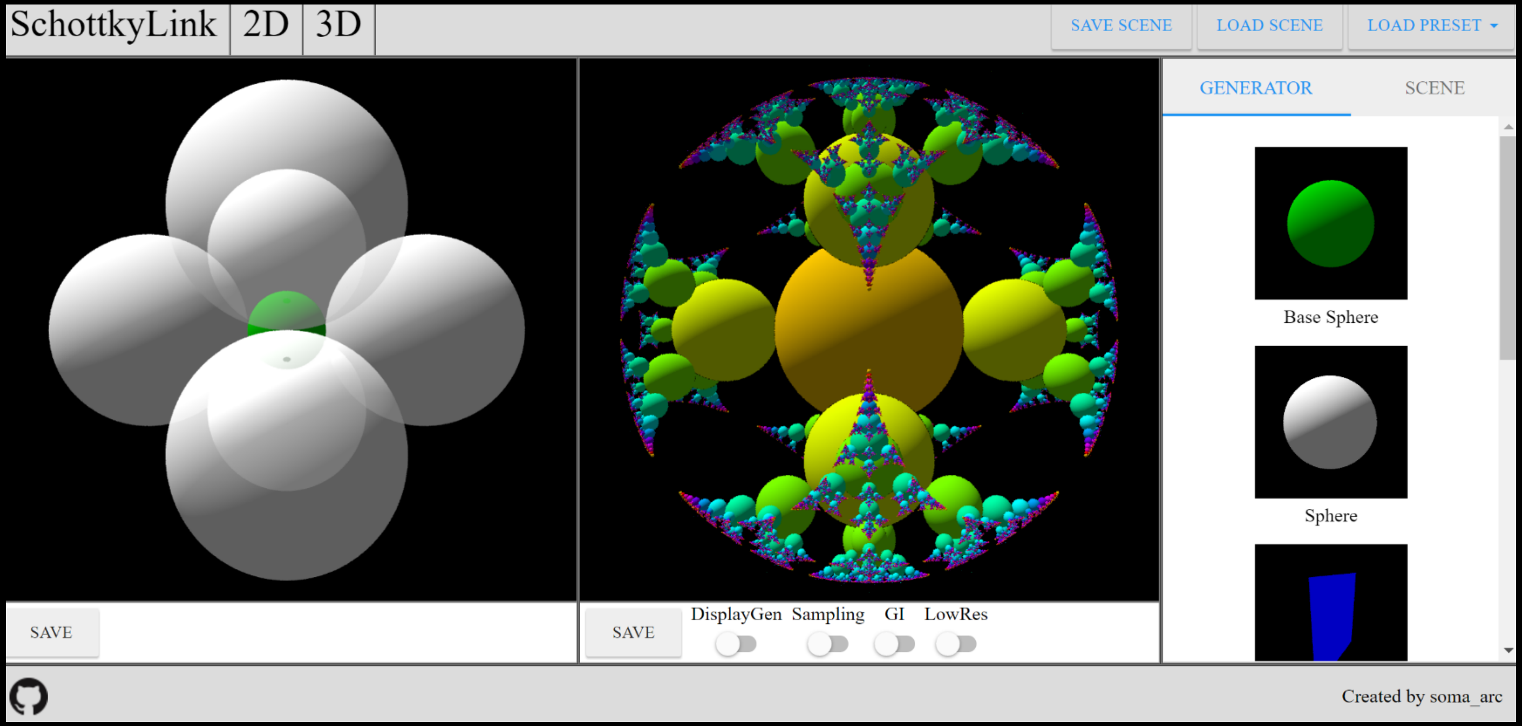
- Slide

  https://speakerdeck.com/soma_arc/a-new-algorithm-for-rendering-kissing-schottky-groups

The software

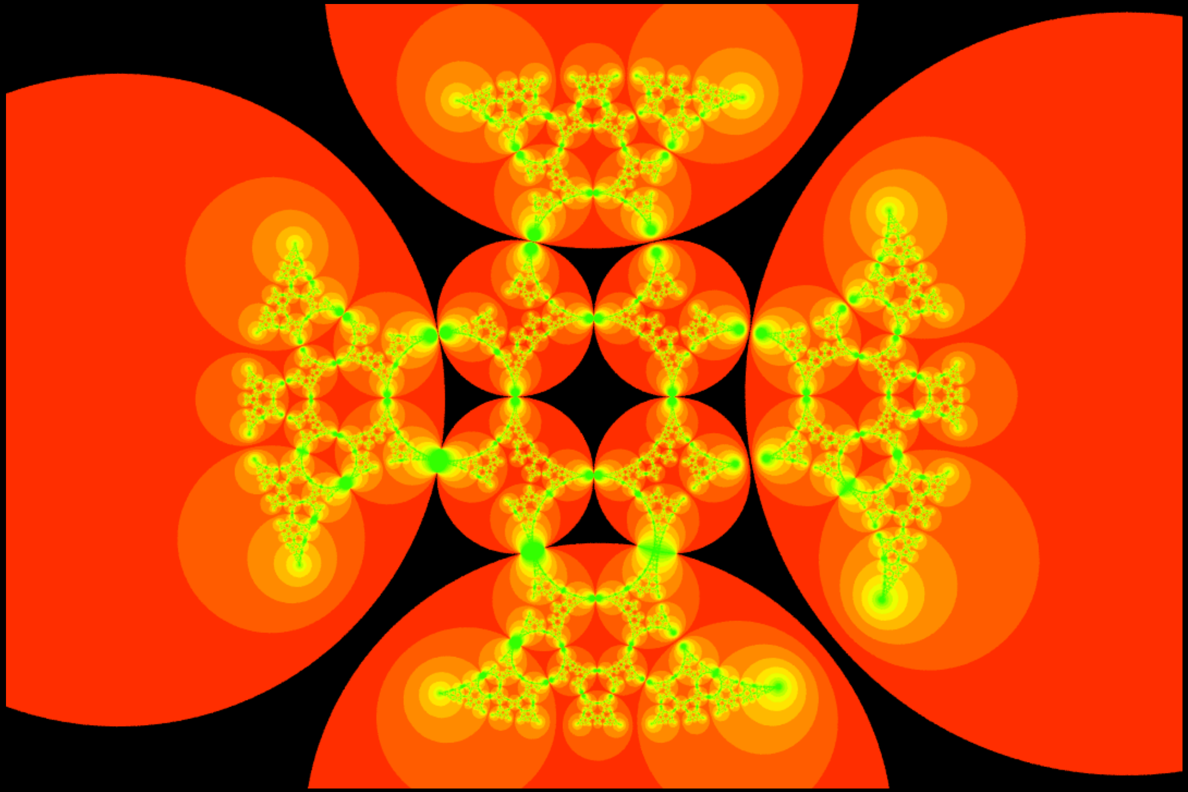
Schottky Link

- Web Application (JavaScript + WebGL)
- URL: schottky.jp
- Source: https://github.com/soma-arc/SchottkyLink
- License : GPL-3.0

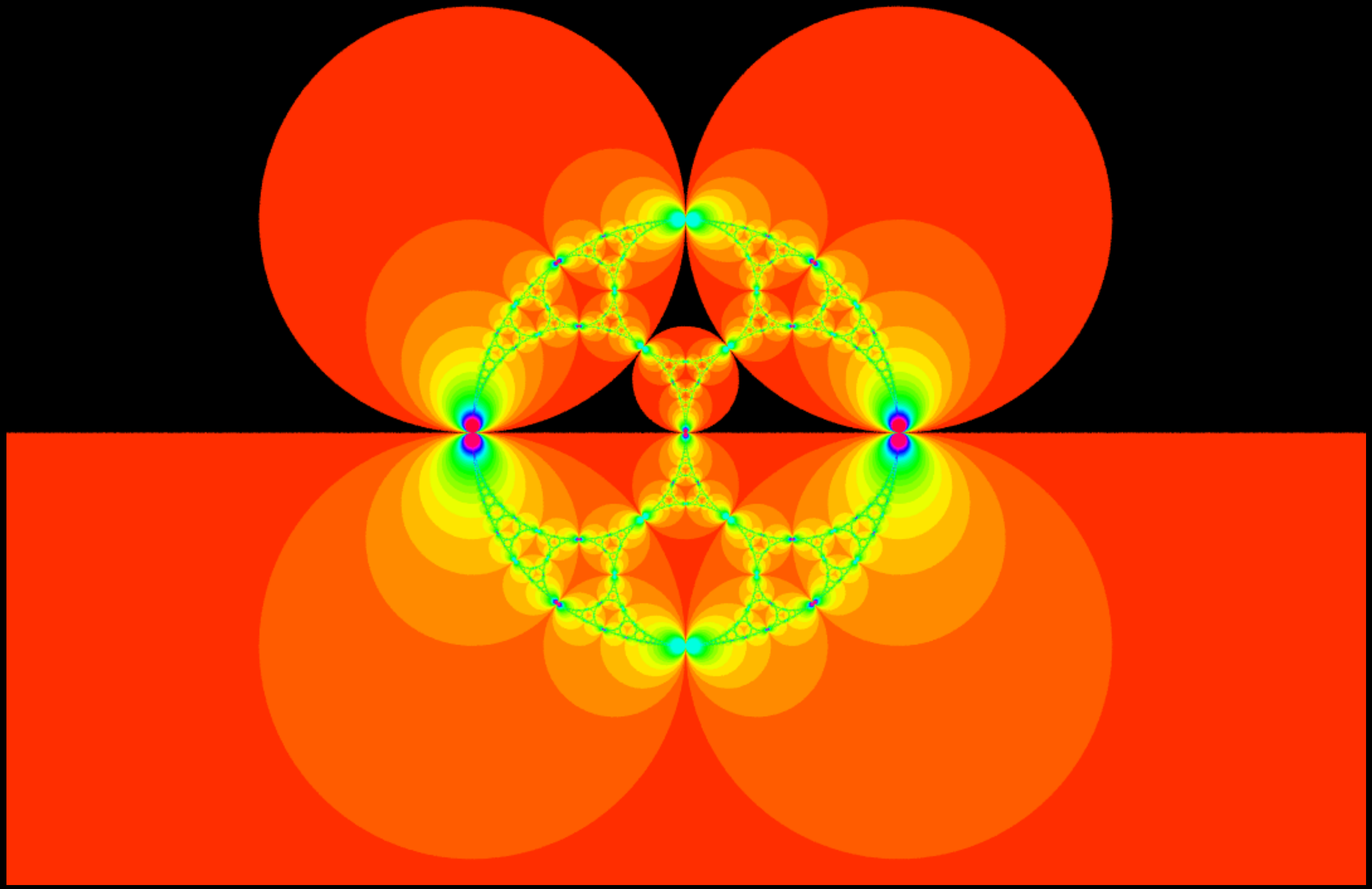


2 Dimensional

Hyperbolic (Loxodromic)

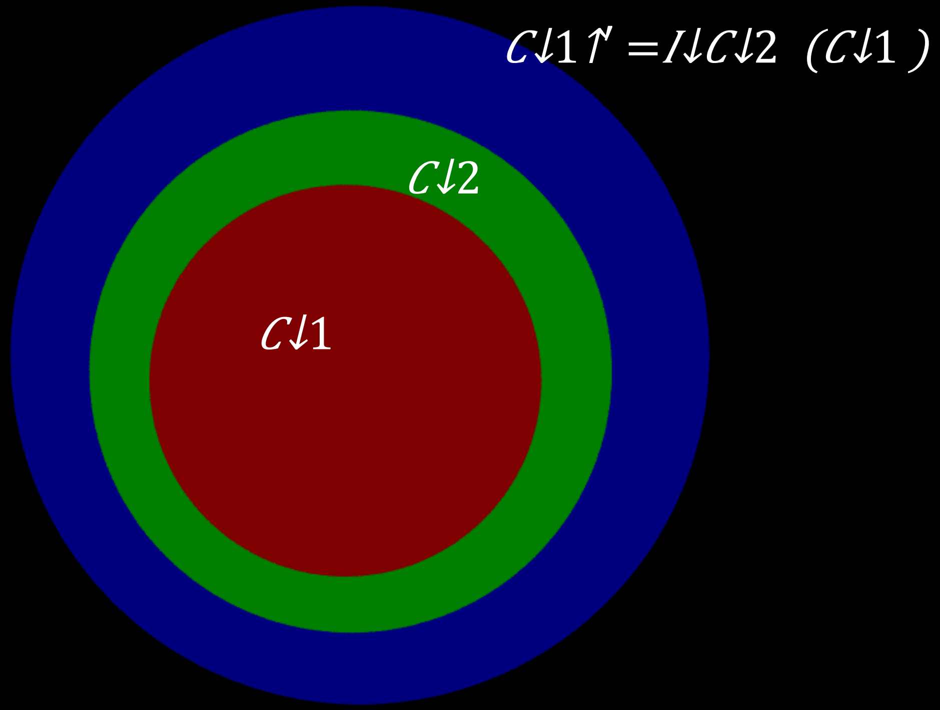


Parabolic (Inversion of Infinite Circle)

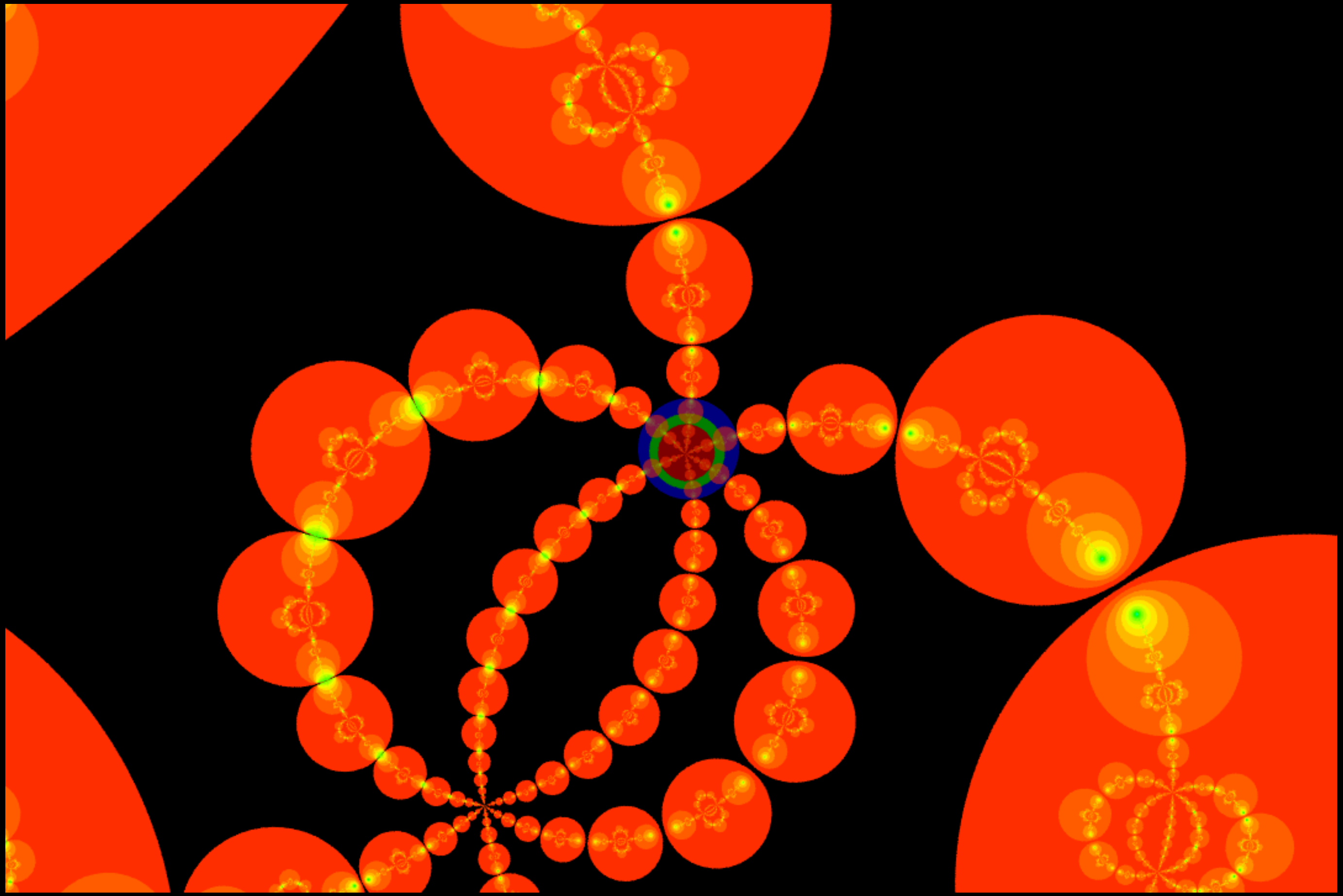


Hyperbolic (Loxodromic)

f p is inside of C_1
pply $I_{C_2} \circ I_{C_1}$
f p is outside of C_1'
pply $I_{C_1} \circ I_{C_2}$

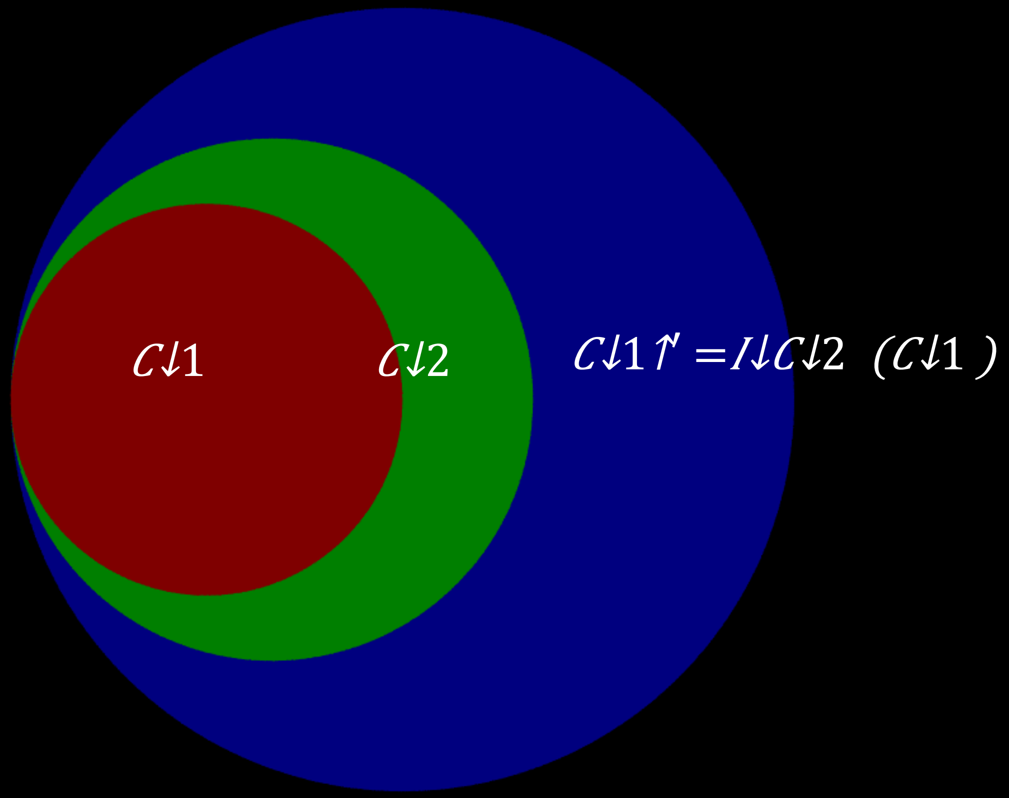


Hyperbolic

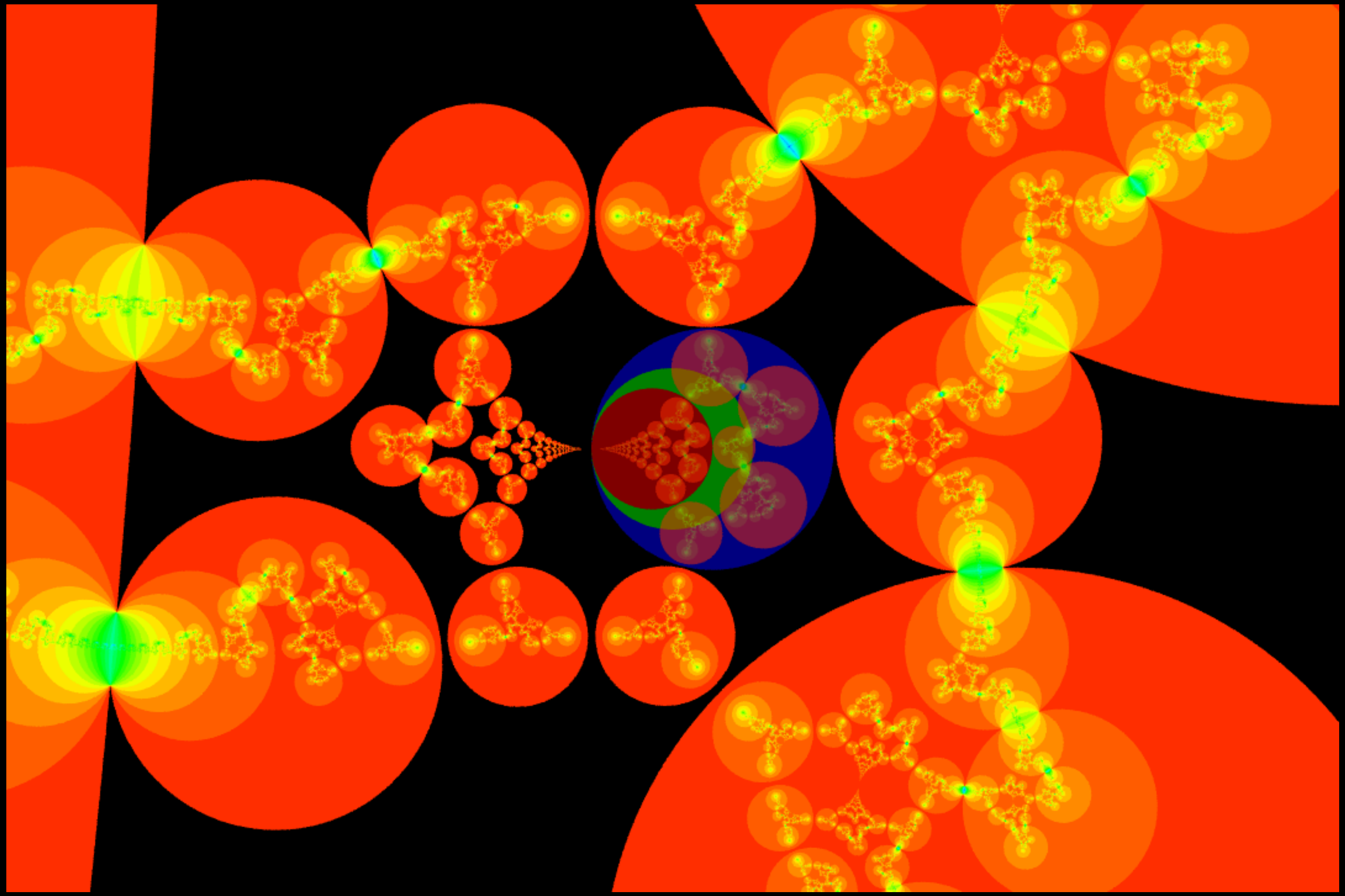


Parabolic

f p is inside of C_1
pply $I_{C_2} \circ I_{C_1}$
f p is outside of C_1'
pply $I_{C_1} \circ I_{C_2}$



Parabolic



Loxodromic

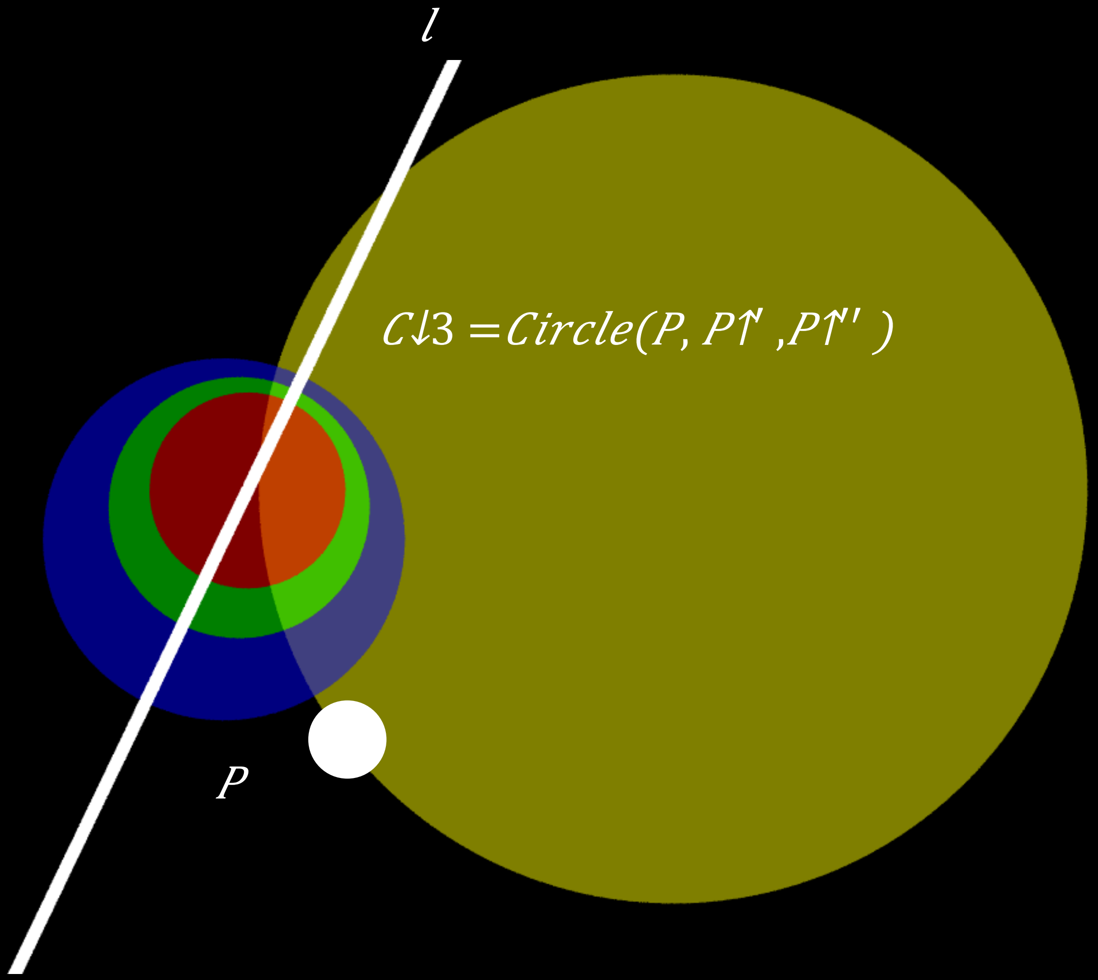
$controlPoint: P$

$P' = I_{C_1}(P)$

$P'' = I_{C_2}(P)$

$l = Line(C_1, C_2)$

$C_3 = Circle(P, P', P'')$



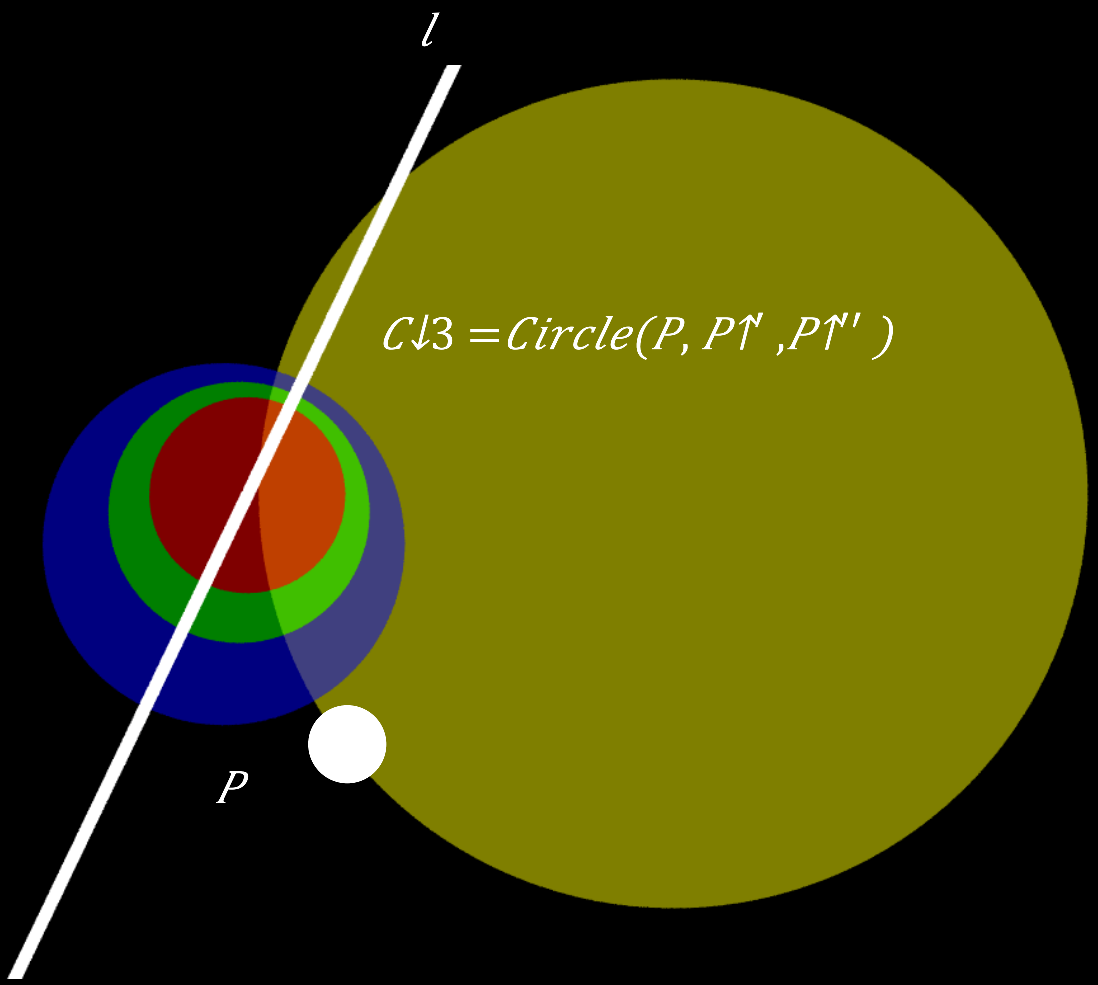
Loxodromic

if p is inside of C_1

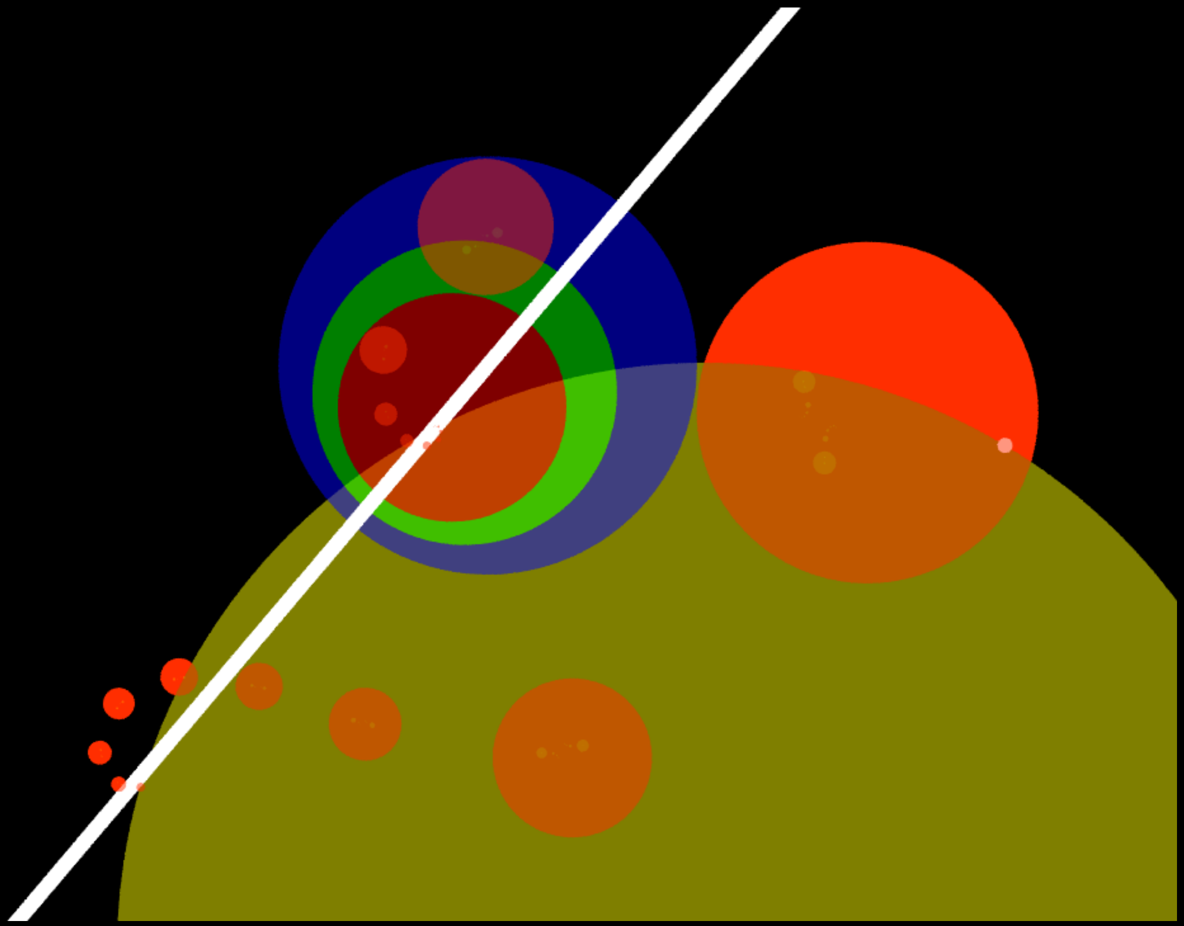
apply $I_{C_2} \circ I_{C_1} \circ I_{C_3} \circ I_l$

if p is outside of C_1'

apply $I_l \circ I_{C_3} \circ I_{C_1} \circ I_{C_2}$

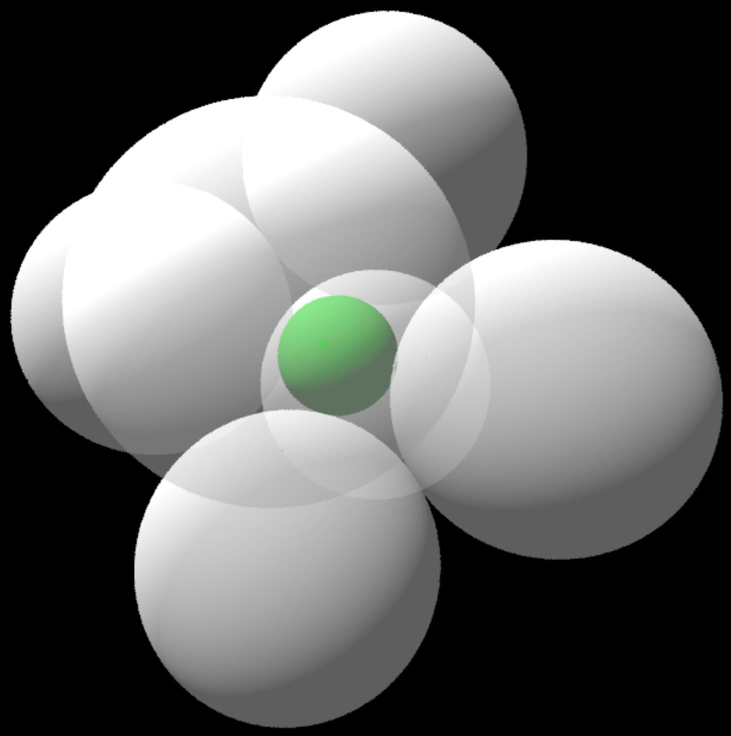


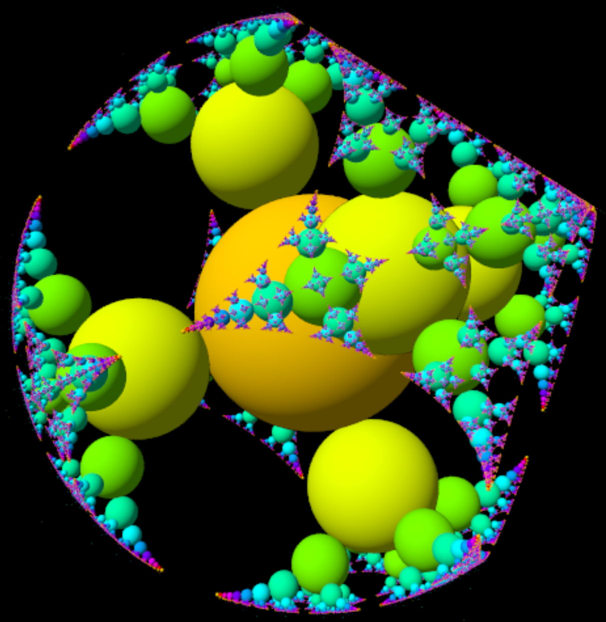
Loxodromic

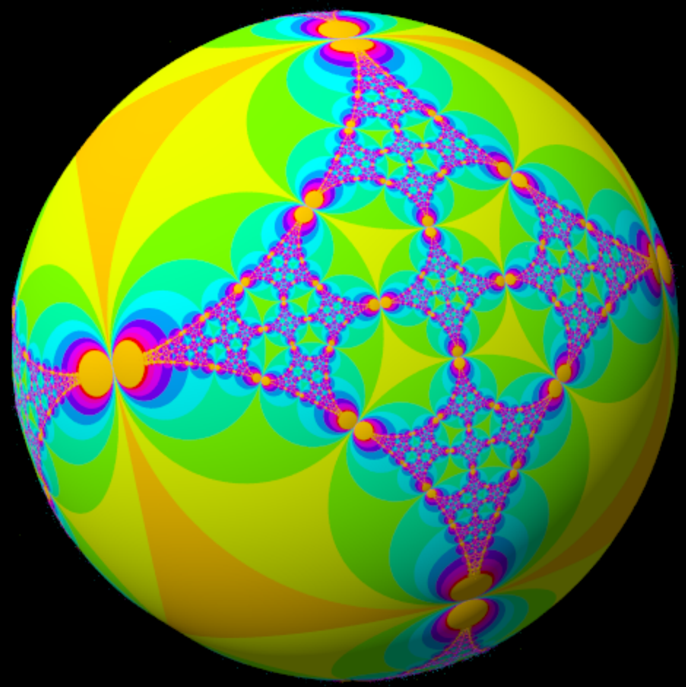


3 Dimensional

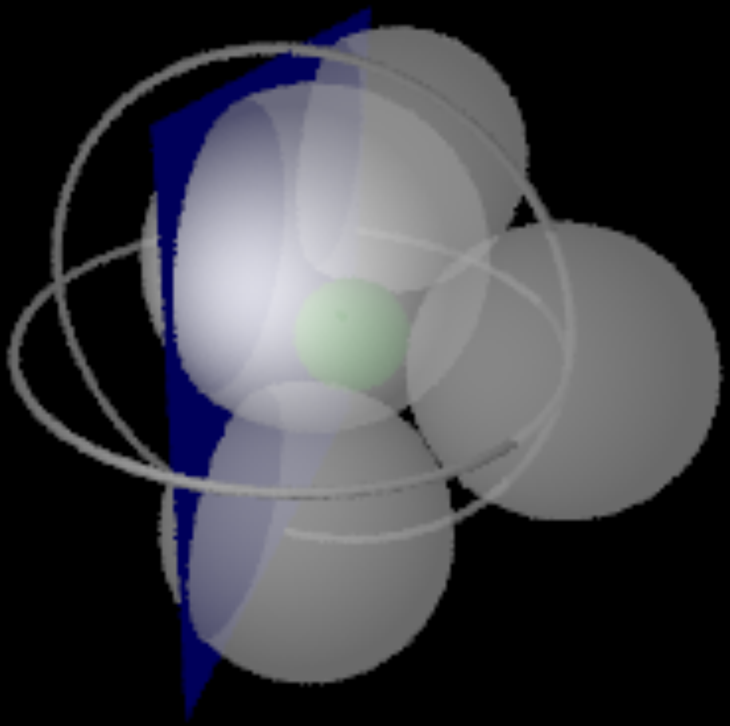
Hyperbolic

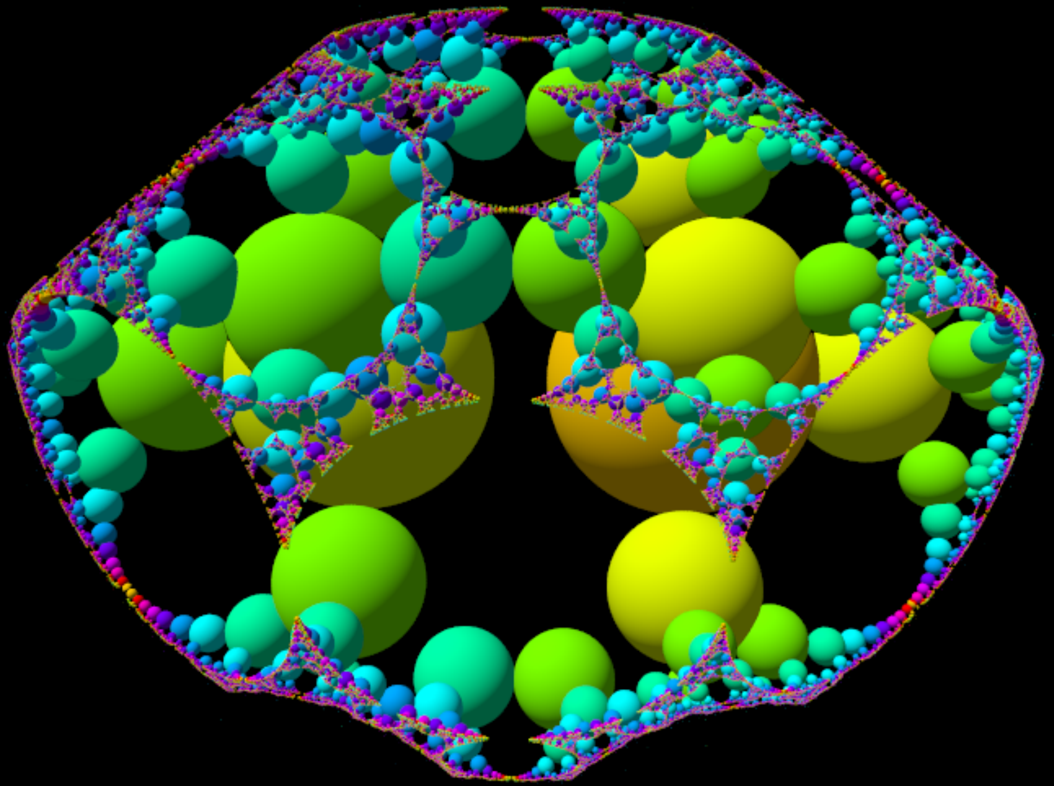




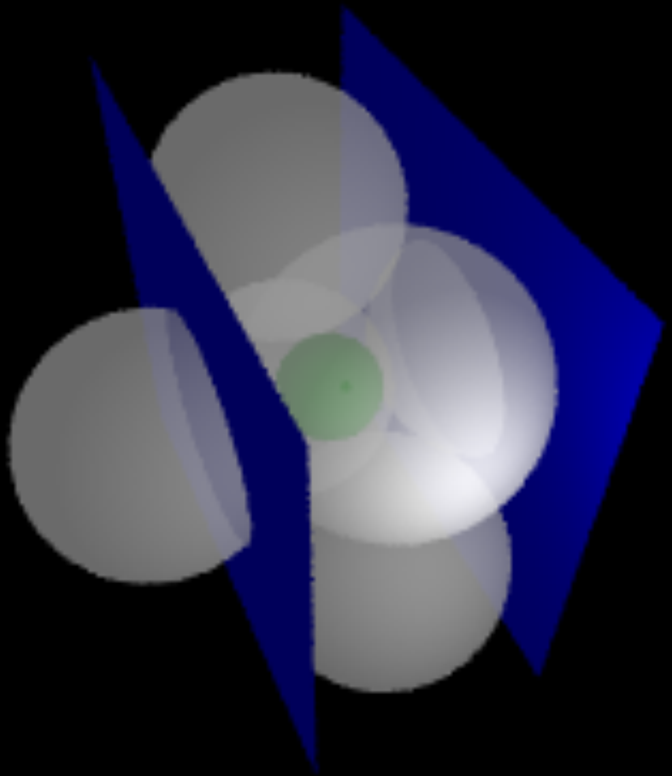


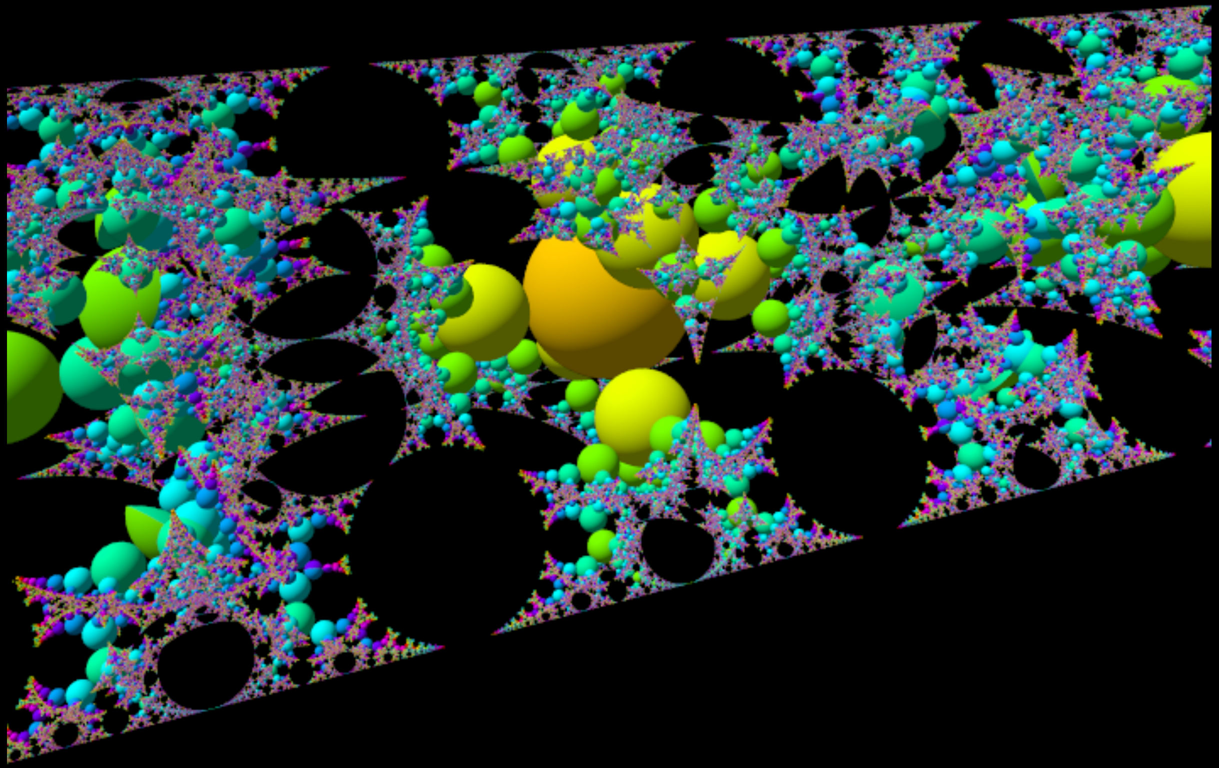
Parabolic (Inversion of Infinite Sphere)



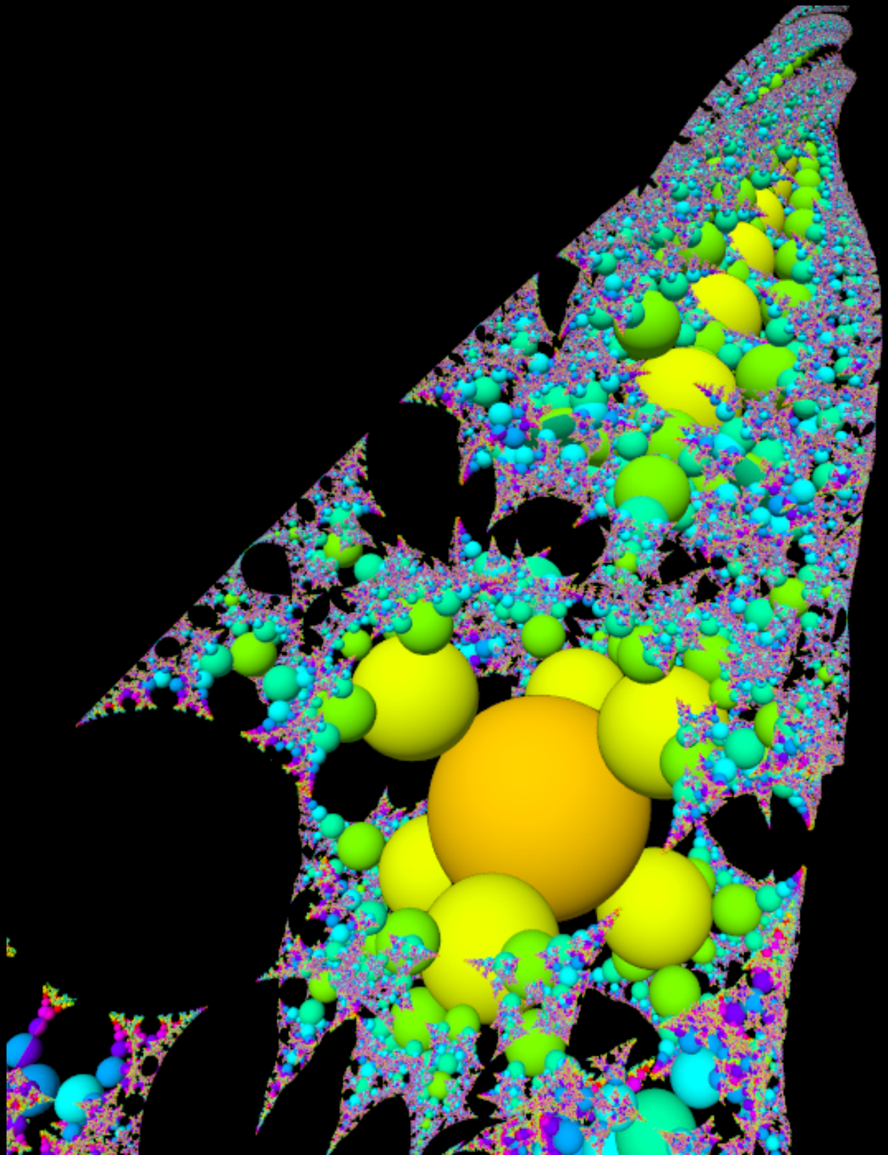


Parabolic (Translation)

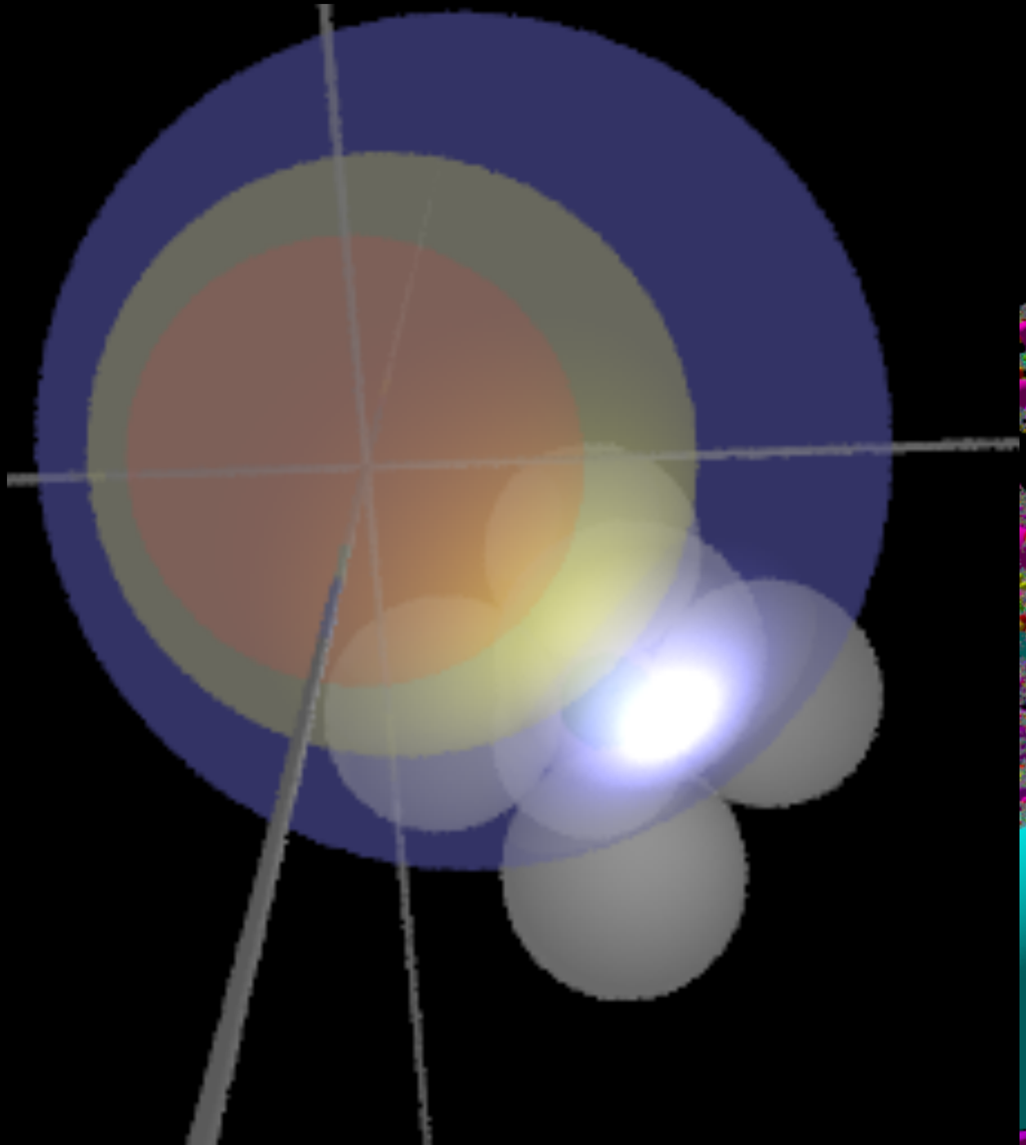


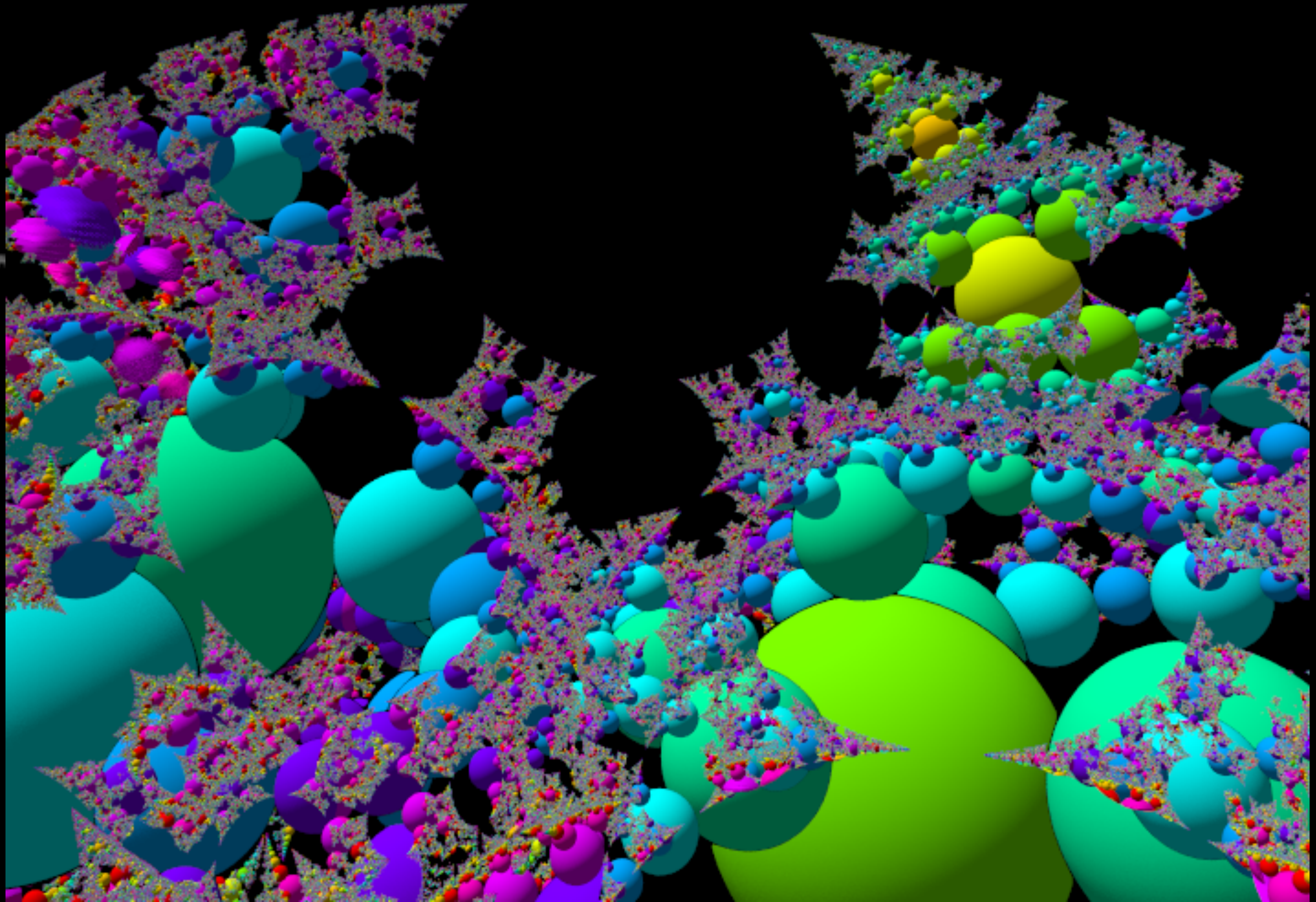


Compound Parabolic (Translation + Rotation)

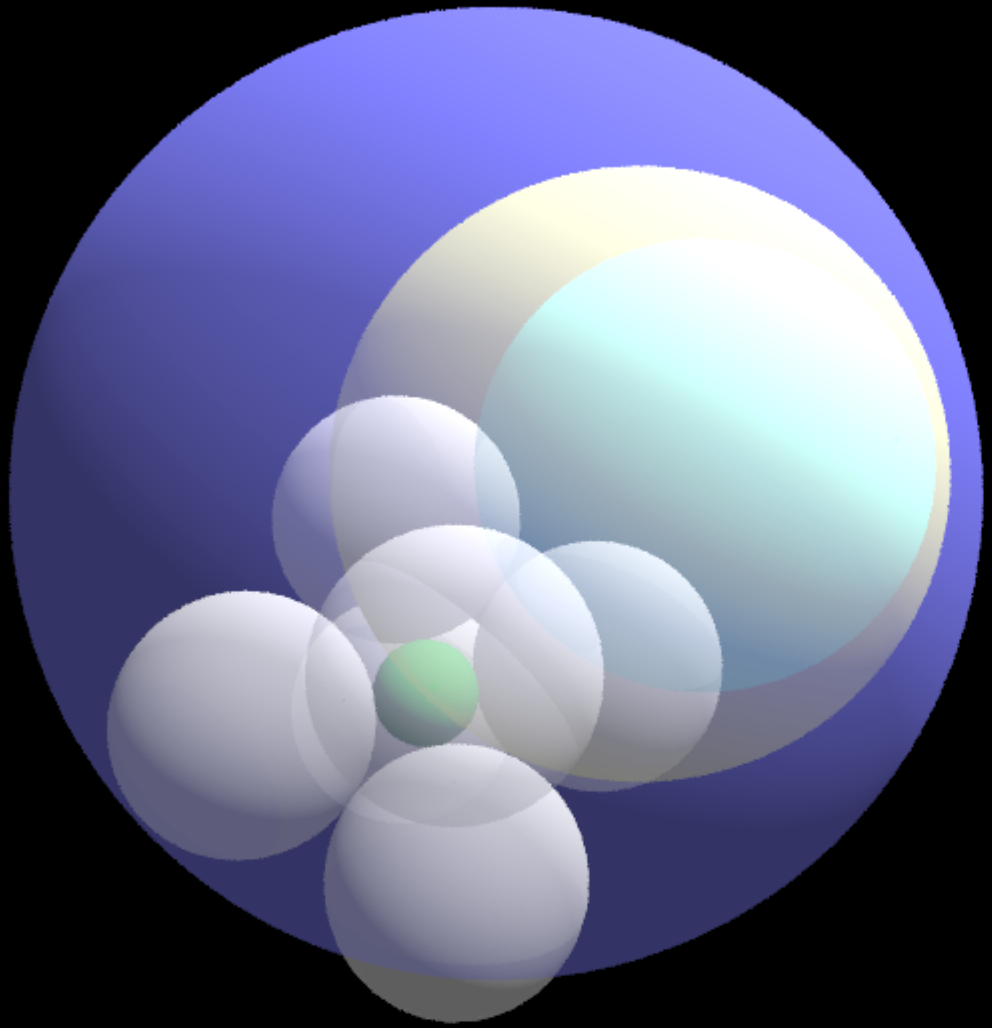


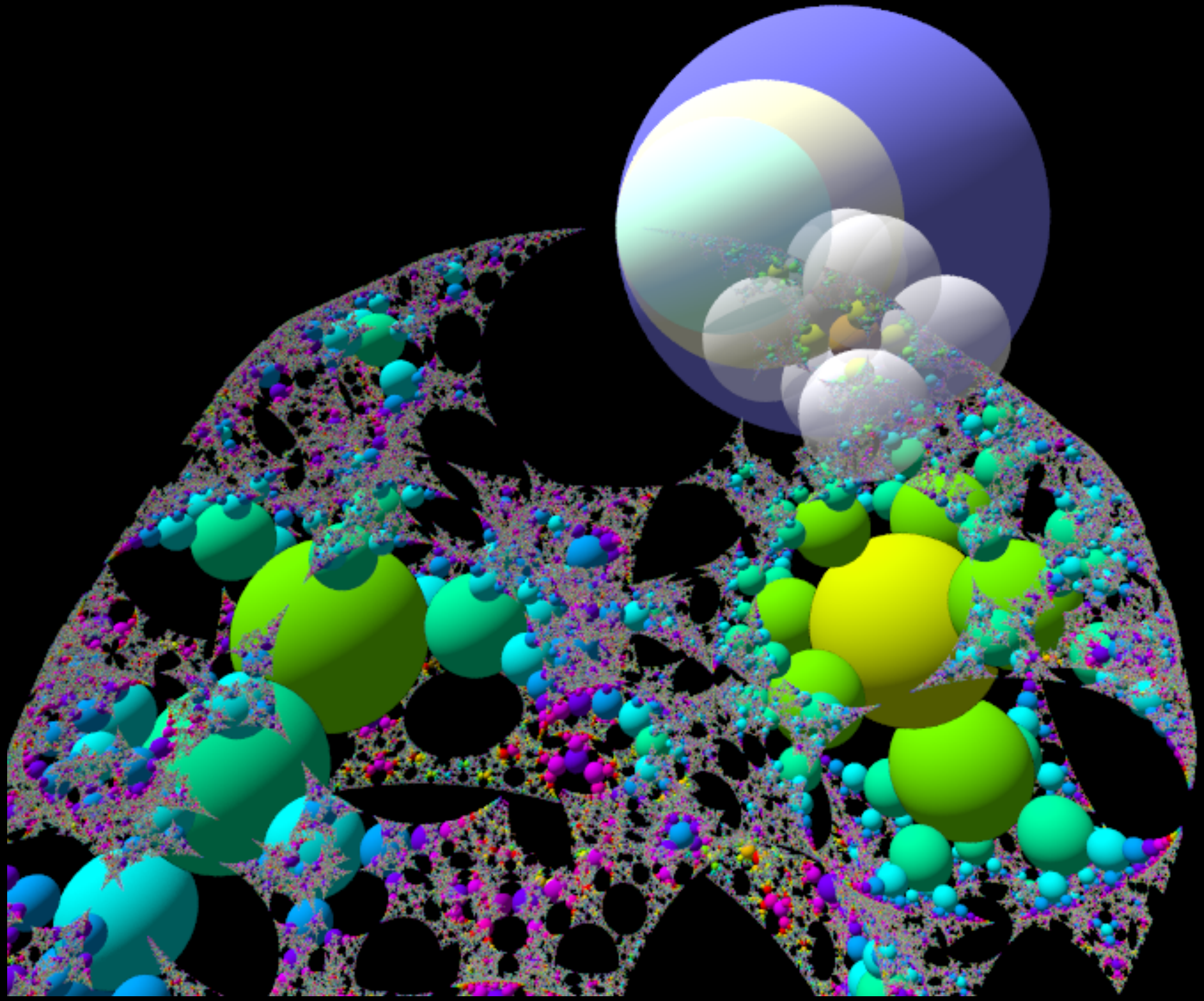
Loxodromic





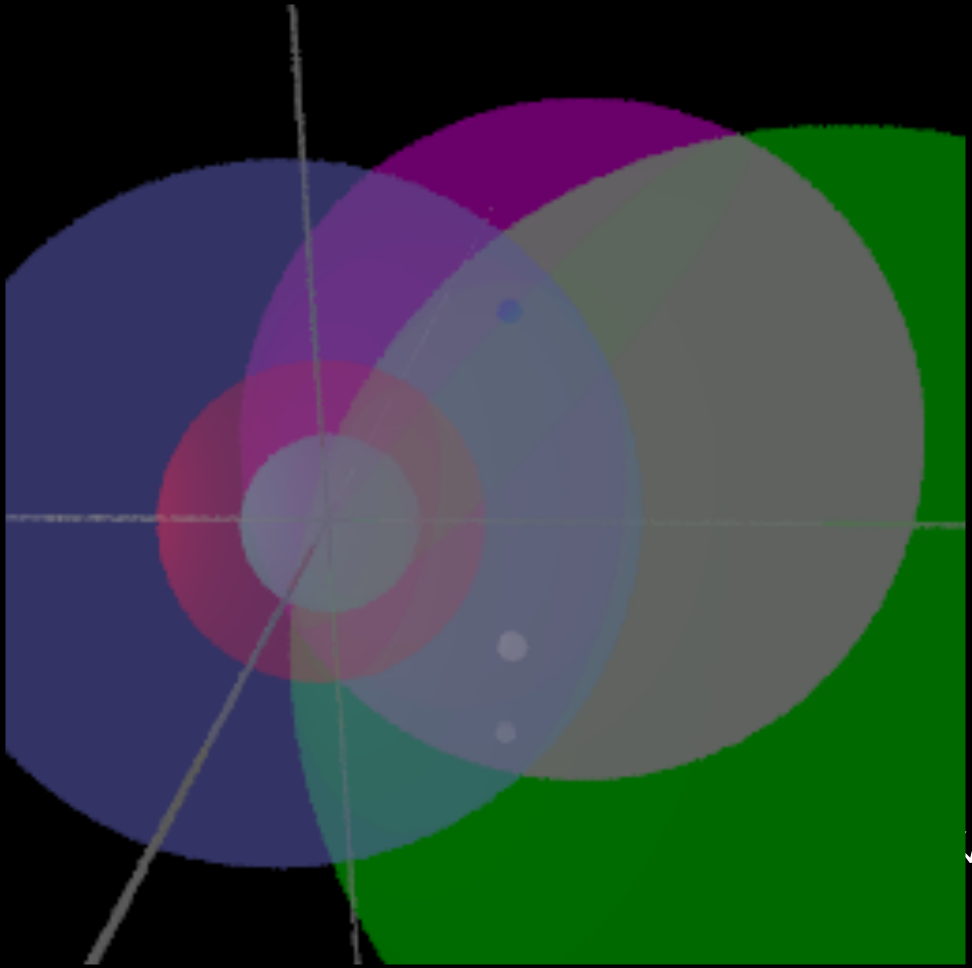
Parabolic





Compound Loxodromic

S_3



↓4

$controlPoint: P, Q_1, Q_2$

$P' = I_{S_1}(P)$

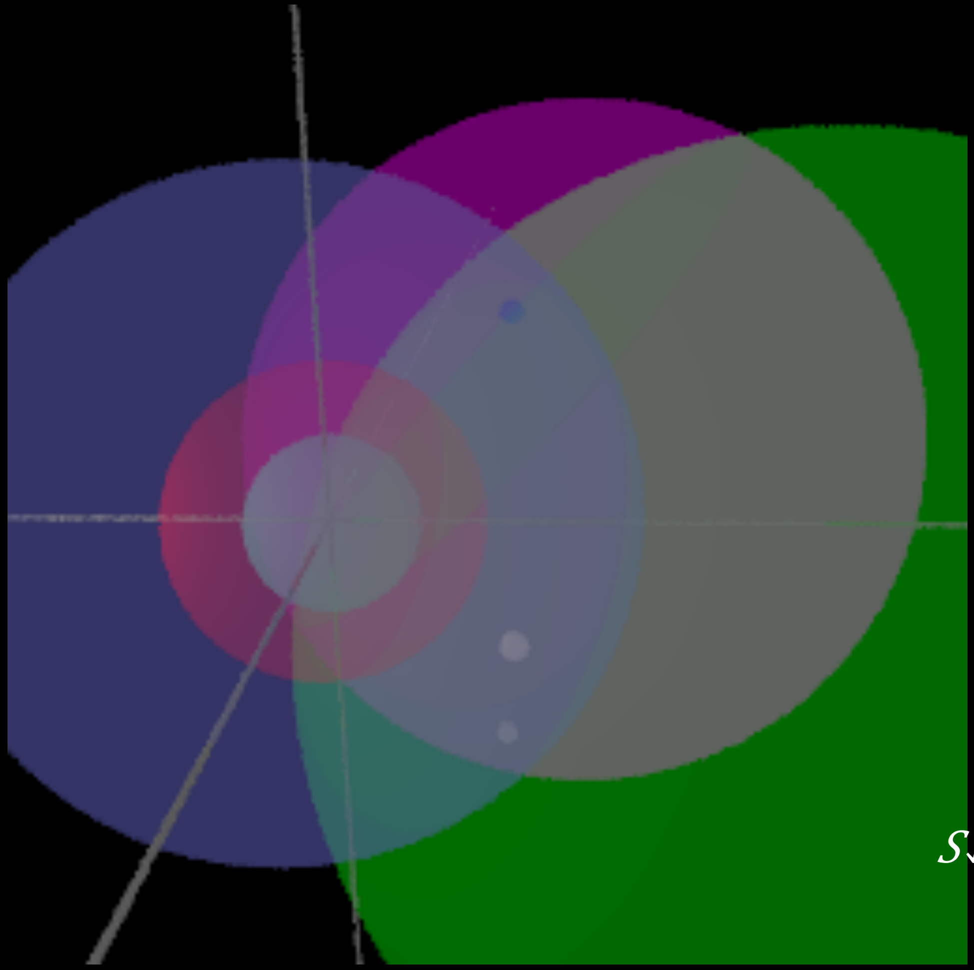
$P'' = I_{S_2}(P)$

$S_3 = Sphere(P, P', P'', Q_1)$

$S_4 = Sphere(P, P', P'', Q_2)$

Compound Loxodromic

S_3



$if\ p\ is\ inside\ of\ S_1$

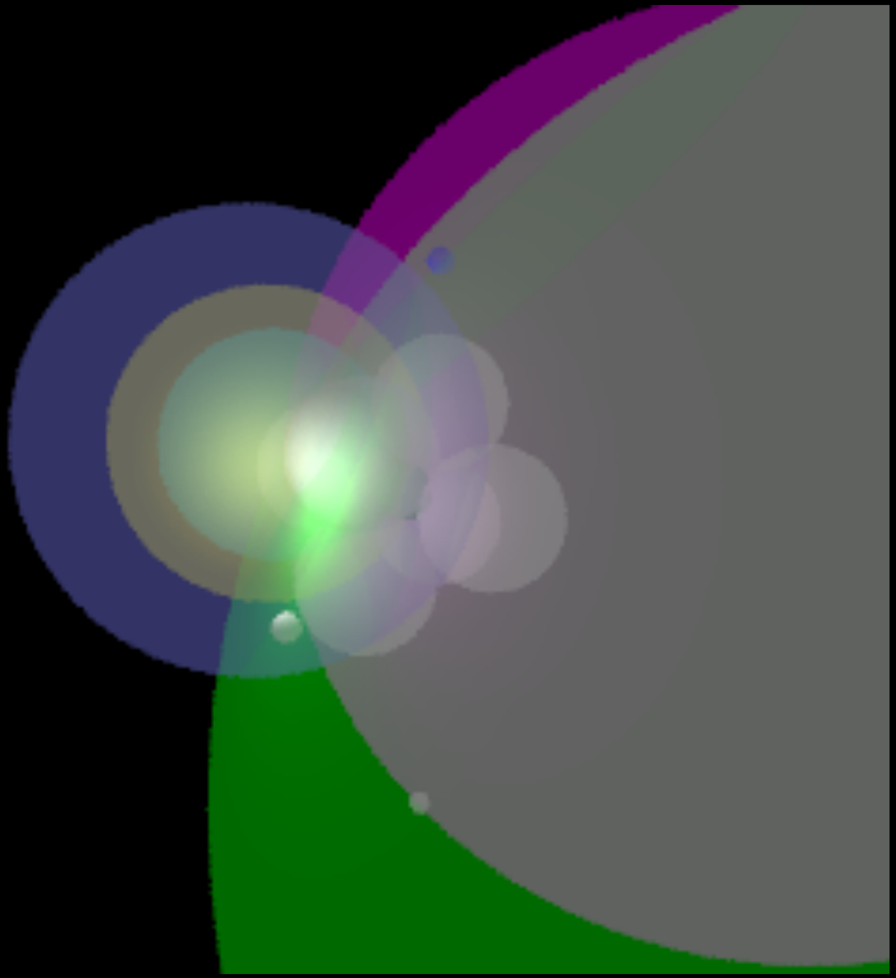
$apply\ I_{S_4} \circ I_{S_3} \circ I_{S_1} \circ I_{S_2}$

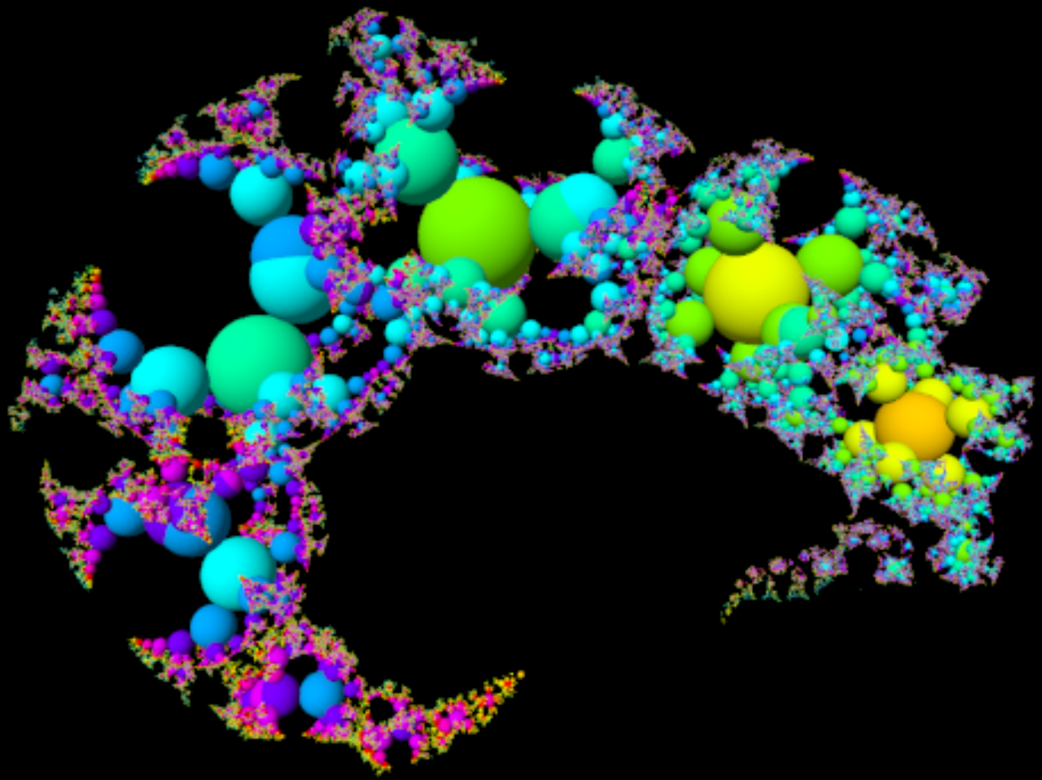
$if\ p\ is\ outside\ of\ S_1'$

$apply\ I_{S_2} \circ I_{S_1} \circ I_{S_3} \circ I_{S_4}$

S_4

Compound Loxodromic





Summary

Iterated Inversion System (IIS)

- Iterate inversion until the point enters the fundamental domain
- Parallelization with Fragment Shader in GLSL
- Render orbit of spheres using raymarching

The software - Schottky Link

- Implement some kind of generators other than simple inversion
- It enables us to visualize complicated Kleinian groups easily
- This system will be helpful to researchers and also fractal artists.
- URL [schottky.jp]
- Source Code on GitHub
  [https://github.com/soma-arc/SchottkyLink]