Experiments on Ford and Dirichlet domains for 3-dimensional cone hyperbolic manifolds

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Workshop Topology and Computer 2017
(at Osaka University)
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A part of this talk is "related" with a joint project with Yasushi Yamashita

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Question When $\Gamma = \langle A, B \rangle \in PSL(2,\mathbb{C})$ is discrete for a given pair of elements $A, B \in PSL(2,\mathbb{C})$?

- $PSL(2,\mathbb{C}) = SL(2,\mathbb{C})/\{\pm I\}$

$$\cong \left\{ \varphi : \hat{\mathbb{C}} \to \hat{\mathbb{C}} \;\middle|\; \varphi(z) = \frac{az+b}{cz+d} \right\}$$

$$\cong \mathrm{Isom}^+ \mathbb{H}^3$$

- $\Gamma < PSL(2,\mathbb{C}) \longleftrightarrow M = \mathbb{H}^3/\Gamma$

Γ : discrete " $\overset{}{\underset{\text{almost}}{\Longleftrightarrow}}$ " M : a complete hyp $\overset{\text{(orbifold)}}{\text{manifold}}$ with $\pi_1(M) \cong \Gamma$

Case $[A,B]$: parabolic

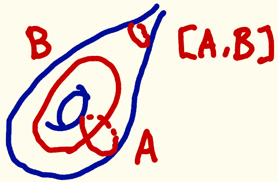
A practical algorithm is known:

INPUT $A, B \in PSL(2,\mathbb{C})$ with $[A,B]$ parab

OUTPUT One of the following:

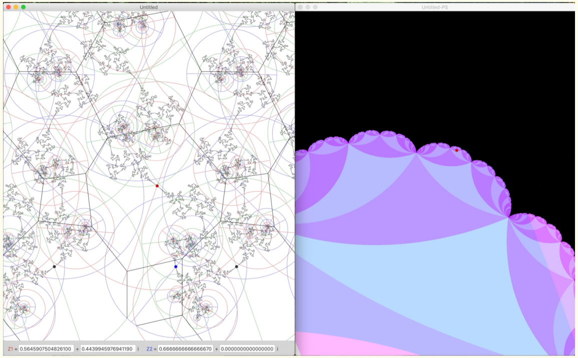
- (i) Γ is free and discrete (Jorgensen theory)
- (ii) Γ is not discrete (Shimizu–Leutbecher)
- (iii) Don't know

Background Quasifuchsian space for

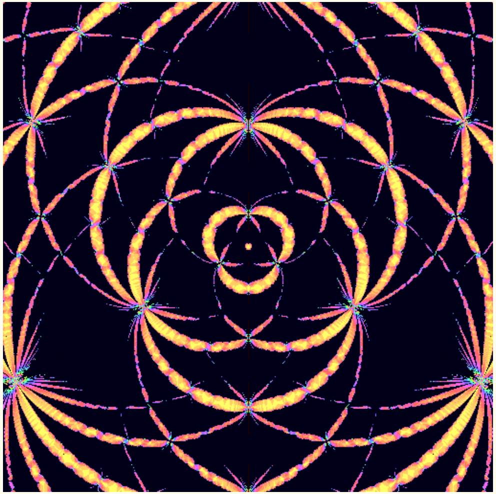


punctured torus

Softwares



OPTi by M. Wada



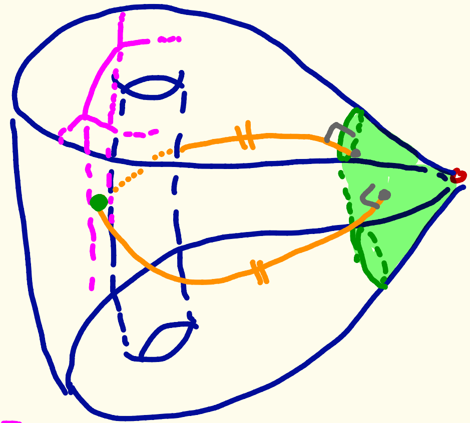
Picture of a Bers slice produced by Yasushi Yamashita

based on his joint work with Y. Komori, T. Sugawa & M. Wada

Jorgensen Theory

gives a characterization of the comb. str. of the Ford domains for punctured torus groups.

* Ford domain P :
  a canonical fund. domain for $\Gamma \curvearrowright \mathbb{H}^3$



$M = \mathbb{H}^3/\Gamma \approx T_0 \times \mathbb{R}$

$\cup$

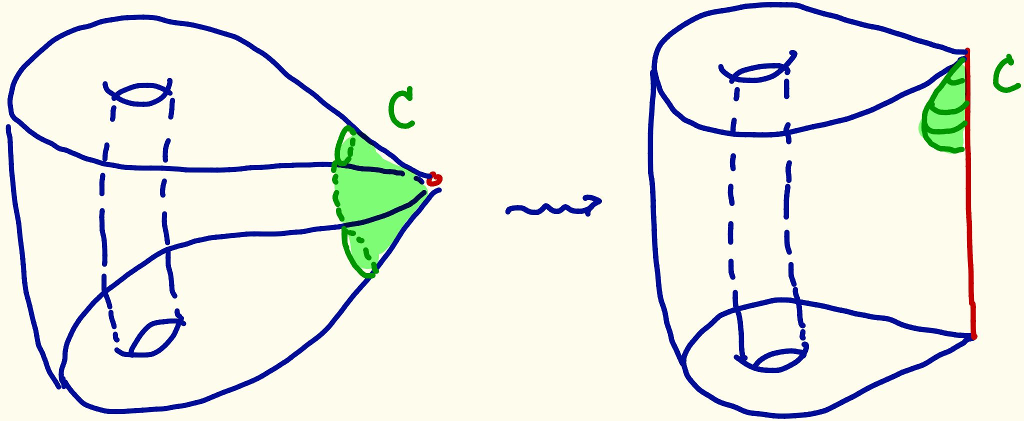
$C = \text{Horoball}/\langle \text{parab} \rangle$: cusp

$P = \overline{M - (\text{cut locus})}$

$\exists$ at least 2 shortest paths

A possible variation of Jorgensen Theory

Change "puncture" to "cone point"

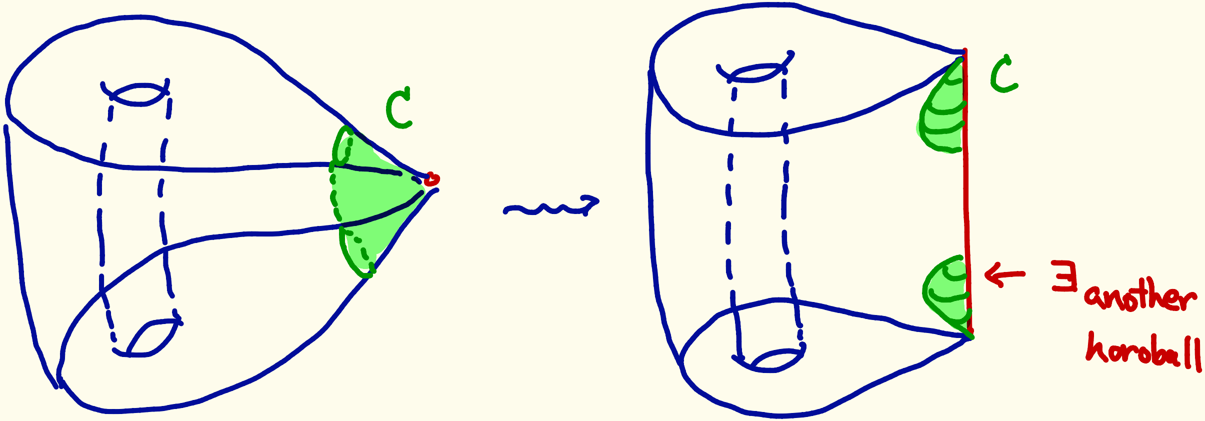


$M = T_0 \times \mathbb{R}$ $\qquad$ $M = T_\theta \times \mathbb{R}$ $(0 < \theta < 2\pi)$

$\longrightarrow$ $\exists$ a natural extension of "Ford domains"

A possible variation of Jorgensen Theory

Change "puncture" to "cone point"



$M = T_0 \times \mathbb{R}$ $\qquad$ $M = T_\theta \times \mathbb{R}$ $(0 < \theta < 2\pi)$

$\rightarrow$ $\exists$ a natural extension of "Ford domains"

BUT NOT CANONICAL!!

6/10

Problems

① The "corresponding" group $\langle A, B \rangle$ is
as the image of hol. repr.
not discrete when $\theta/\pi \notin \mathbb{Q}$.

①′ $\nexists$ canonical universal covering (like $\mathbb{H}^3$).

② The "horoballs" are NOT unique.

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→ Some argument using CAT(−1) space.

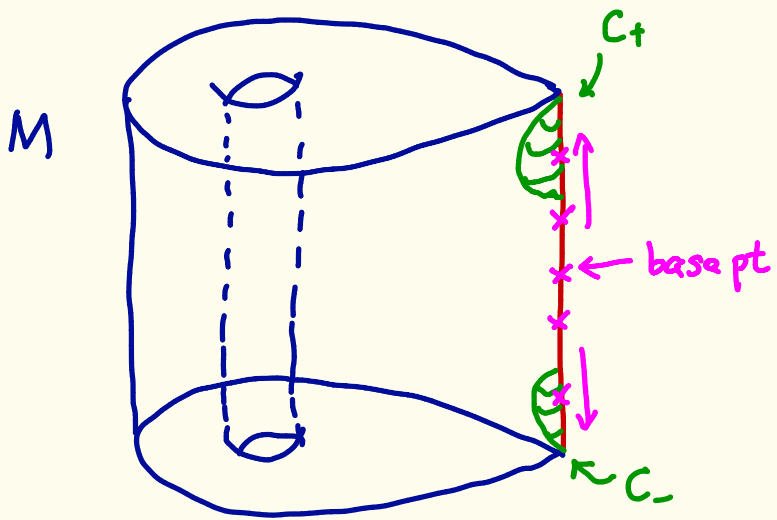
② The "horoballs" are NOT unique.

→ Connect them by the family of Dirichlet domains. (Today's experiment.)

Ford and Dirichlet domains

Recall: Ford domain = $\overline{M - (\text{cut locus w.r.t. } C)}$

Def Dirichlet domain = $\overline{M - (\text{cut locus w.r.t. base pt})}$



Thm If M contains a strongly convex compact subset, then $\#\{\text{comb. str. of Ford \& Dirichlet domains}\} < \infty$.

A software (under construction)

- Cone angle is fixed.
- Can move
  - hyperbolic structures
  - base points for Dirichlet domains
- Looks like (a very early version of) OPTi
- Developed with Apple's "swift" and runs on my MacBook Pro.

→ Let's see....

Questions (1/2) — in mathematics

- It is likely that:
  If we order to the software "draw limit set", then it will draw a Jordan curve on "the plane."
  What does it mean?

- Is there any correspondance between "the plane" and the imaginary boundary of a reasonable space?

Questions (2/2) — in programing

How can I improve the speed?

My software uses "SceneKit" of "Cocoa".

- Can I produce "real-time" views?

  The scene contains only <1000 hemispheres.

- Suggestions like

  "Use another development environment!"

  is also welcome.

Thank you